Online Energy Generation Scheduling for Microgrids with Intermittent Energy Sources and Co-Generation

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ABSTRACT

Microgrids represent an emerging paradigm of future electric power systems that can utilize both distributed and centralized generations. Two recent trends in microgrids are the integration of local renewable energy sources (such as wind farms) and the use of co-generation (*i.e.,* to supply both electricity and heat). However, these trends also bring unprecedented challenges to the design of intelligent control strategies for the microgrids. Traditional generation scheduling paradigms assuming perfect prediction of future electricity supply and demand are no longer applicable to microgrids with unpredictable renewable energy supply and co-generation (that needs to conside both electricity and heat demand). In this paper, we study online algorithms for the micro-grid generation scheduling problem with intermittent renewable energy sources and co-generation, in order to maximize the cost-savings with local generation. Based on the insights from the structure of the offline optimal solution, we propose a class of competitive online algorithms, called CHASE (Competitive Heuristic Algorithm for Scheduling Energy-generation), that track the offline optimal in an online fashion. Under certain settings, we show that CHASE achieves the best competitive ratio of all deterministic online algorithms. We also extend our algorithms to intelligently leverage on *limited prediction* of the future, such as near-term demand or wind forecast. By extensive empirical evaluation using real-world traces, we show that our proposed algorithms can achieve near-offline-optimal performance. In a representative scenario, CHASE leads to around 20% cost reduction with no future look-ahead, and the cost reduction increases with the future look-ahead window.

1. INTRODUCTION

Microgrid is a distributed electricity system that can autonomously co-ordinates local generations and demands in a dynamic manner [23]. Illustrated in Fig. 1, modern microgrids often consist of distributed renewable energy generations (*e.g.,* wind) and co-generation technology (*e.g.,* supplying both electricity and heat locally). Microgrids can operate in either grid-connected mode or islanded mode. There have been worldwide deployments of pilot microgrids, such as US, Japan, Greece and Germany [7].

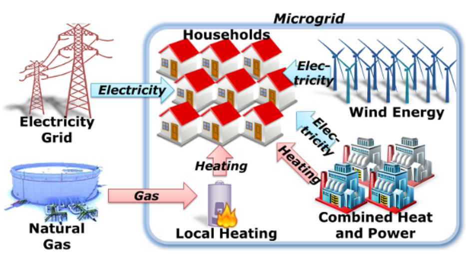

Figure 1: An illustration of microgrid.

Microgrids are more robust and cost-effective than traditional approach of centralized grid. They represent an emerging paradigm of future electric power systems [26] that address the following two critical challenges.

(1) *Power Reliability.* Providing reliable and quality power is critical both socially and economically. In the US alone, while the electricity system is 99.97% reliable, each year the economic loss due to power outage is at least $150 billion [29]. However, enhancing power reliability across a large-scale power grid is very challenging [13]. With local generation, microgrids can supply energy *locally* as needed, effectively alleviating the negative effects of power outage.

(2) *Integration with Renewable Energy.* The growing environmental awareness and government directives lead to the increasing penetration of renewable energy. For example, United States aims at 20% wind energy penetration by 2030 to "de-carbonize" the power system. Denmark targets at 50% wind generation by 2025. However, incorporating a significant portion of intermittent renewable energy poses great challenges to grid stability, which requires a new thinking of how the grid should operate [37]. In traditional centralized grid, the actual locations of conventional energy generation,

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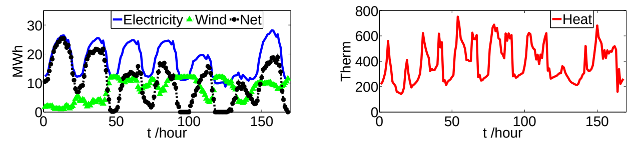
renewable energy generation (*e.g.,* wind farms), and energy consumption are usually distant from each other. Thus, the need to coordinate conventional energy generation and consumption based on the instantaneous variations of renewable energy generation leads to challenging stability problems. In contrast, in microgrids renewable energy is generated and consumed in the *local* distribution network. Thus, the uncertainty of renewable energy is absorbed locally, minimizing its negative impact on the stability of the central transmission networks.

Furthermore, microgrids bring significant economic benefits, especially with the augmentation of combined heat and power (CHP) generation technology. In traditional grids, a substantial amount of residual energy after electricity generation is often wasted. In contrast, in microgrids this residual energy can be used to supply heat domestically. By simultaneously satisfying electricity and heat demand using CHP generators, microgrids can often be much more economical than obtaining electricity from the centralized grid [18].

However, to realize the maximum benefits of microgrids, intelligent scheduling of both local generation and demand must be established. Dynamic demand scheduling in response to supply condition, also called *demand response* [29, 12], is one of the useful approaches. But, demand response alone may be insufficient to compensate the highly volatile fluctuations of wind generation. Hence, intelligent generation scheduling is indispensable for the viability of microgrids. Such generation-side scheduling must simultaneously meet two goals. (1) To maintain grid stability, the aggregate supply from CHP generation, renewable energy generation, the centralized grid, and a separate heating system must meet the aggregate electricity and heat demand. (We do not consider the option of using energy storage in the paper, e.g., to charge at low-price periods and to discharge at the high-price periods. This is because for the typical size of microgrids, *e.g,* a college campus, energy storage systems with comparable sizes are very expensive and not widely available.) (2) It is highly desirable that the microgrid can coordinate local generation and external energy procurement to minimize the overall cost of meeting the energy demand.

We note that a related generation scheduling problem has been extensively studied for the traditional grid, involving both Unit Commitment (UC) [30] and Economic Dispatch (ED)[1] [14]. In a typical power plant, the generators are often subject to severe operational constraints. For example, steam turbines have a long startup time and a slow ramp-up speed. In order to perform generation scheduling, the utility company usually needs to forecast the demand first. Based on this forecast, the utility company then solves an offline problem to schedule different types of generation sources in order to minimize cost subject to the operations constraints.

Unfortunately, this classical strategy does not work well for the microgrids due to the following unique challenges introduced by the renewal energy sources and co-generation. The first challenge is that microgrids powered by intermittent renewable energy generations will face a significant uncertainty in energy supply. Because of its smaller scale, abrupt changes in local weather condition may have a dramatic impact that cannot be amortized as in the wider national scale. In Fig. 2a, we examine one-week traces of electricity demand for a college in San Francisco [1] and power output of a nearby wind station [4]. We observe that although the electricity demand has a relative regular pattern for prediction, the net electricity demand inherits a large degree of variability from the wind generation, casting a challenge for accurate prediction.



(a) Elec. demand and wind gen.

(b) Heat demand

Figure 2: Electricity demand, heat demand and wind generation in a week. In (a), the net demand is computed by subtracting the wind generation from the electricity demand.

Secondly, co-generation brings a new dimension of uncertainty in scheduling decisions. Observed from Fig. 2b, the heat demand exhibits a different stochastic pattern that adds difficulty to the prediction of overall energy demand.

Due to the above additional variability, traditional energy generation scheduling based on *offline* optimization assuming *accurate* prediction of future supplies and demands cannot be applied to the microgrid scenarios. On the other hand, there are also new opportunities. In microgrids there are usually only 1-2 types of small reciprocate generators from tens of kilowatts to several megawatts. These generators are typically gas or diesel powered and can be fired up with large ramping-up/down level in the order of minutes. For example, a diesel-based engine can be powered up in 1-5 minutes and has a maximum ramp up/down rate of 40% of its capacity per minute [38]. The "fast responding" nature of these local generators opens up opportunities to increase the frequency of generator on/off scheduling that substantially changes the design space for energy generation scheduling.

Because of these unique challenges and opportunities, it remains a open question of how to design effective strategies for scheduling energy generation for microgrids.

1.1 Our Contributions

In this paper, we formulate a general problem of energy generation scheduling for microgrids. Since both the future demands and future renewable energy generation are difficult to predict, we use competitive analysis and study online algorithms that can perform provably well under arbitrarily time-varying (and even adversarial) future trajectories of demand and renewable energy generation. Towards this end, we design a class of simple and effective strategies for energy generation scheduling named CHASE (in short for Competitive Heuristic Algorithm for Scheduling Energy-generation). Compared to traditional prediction-based and offline optimization approaches, our online solution has the following salient benefits. First, CHASE gives absolute performance guarantee in any settings without knowledge of supply and demand behaviors. This minimizes the impact of inaccurate modeling and the need for expensive data gathering, and hence improves robustness in microgrid operations. Second,

[1] UC optimizes the startup and shutdown schedule of power generations to meet the forecasted demand over a short period, whereas ED allocates the system demand and spinning reserve capacity among operating units at each specific hour of operation without considering startup and shutdown of power generations.

CHASE works without any assumptions on gas/electricity prices and policy regulations. This provides the grid operators flexibility for operations and policy design without affecting the energy generation strategies for microgrids.

We summarize the key contributions as follows:

1. In Sec. 3.1.1, we devise an optimal offline algorithm for a generic formulation of energy generation scheduling that models most microgrid scenarios with intermittent energy sources and fast-responding gas-/diesel-based CHP generators. Note that even the offline problem is challenging to solve because it is a mix-integer problem and the objective function values across different slots are correlated via the startup cost. We first reveal an elegant structure of the single-generator problem and exploit it to construct the optimal offline solution. The structural insights are further generalized in Sec. 3.3 to the case with N homogeneous generators. The optimal offline solution employs a simple load-dispatching strategy where each generator separately solves a partial scheduling problem.
2. In Secs. 3.1.2-3.3, we build upon the structural insights from the offline solution to design CHASE, a deterministic online algorithm for scheduling energy generations in microgrids. We name our algorithm CHASE because it tracks the offline optimal solution in an online fashion. We show that CHASE achieves a competitive ratio of $\min(3-2\alpha, 1/\alpha) \leq 3$. In other words, no matter how the demand, renewable energy generation and grid prices vary, the cost of CHASE without any future information is guaranteed to be no greater than $\min(3-2\alpha, 1/\alpha)$ times the offline optimal assuming complete future information. Here the constant $\alpha = (c_o + c_m/L)/(P_{\max} + \eta \cdot c_g) \in (0, 1]$ captures the maximum price discrepancy between using local generation and external sources to supply energy. We also prove that the above competitive ratio is the best possible for any deterministic online algorithms.
3. The above competitive ratio is attained without any future information of demand and supply. In Sec. 3.2, we then extend CHASE to intelligently leverage limited look-ahead information, such as near-term demand or wind forecast, to further improve its performance. In particular, CHASE achieves an improved competitive ratio of $\min(3 - 2 \cdot g(\alpha, \omega), 1/\alpha)$ when it can look into a future window of size ω. Here, the function $g(\alpha, \omega) \in [\alpha, 1]$ captures the benefit of looking-ahead and monotonically increases from α to 1 as ω increases. Hence, the larger the look-ahead window, the better the performance. In Sec. 4, we also extend CHASE to the case where generators are governed by several additional operational constraints (*e.g.,* ramping up/down rates and minimum on/off periods), and derive an upper bound for the corresponding competitive ratio.
4. In Sec. 5, by extensive evaluation using real-world traces, we show that our algorithm CHASE can achieve satisfactory empirical performance and is robust to look-ahead error. In particular, a small look-ahead window is sufficient to achieve offline-optimal performance. Our offline (resp., online) algorithm achieves a cost reduction of 22% (resp., 17%) with CHP technology. The cost reduction is computed in comparison with the baseline cost achieved by using only the wind generation, the central grid, and a separate heating system. The cost reductions are relatively significant and show the economic benefit of microgrids in addition to its potential in improving energy reliability. Furthermore, interestingly, deploying a partial local generation capacity that provides 50% of the peak local demands can achieve 90% of the cost reduction. This provides strong motivation for microgrids to deploy at least a partial local generation capability to save costs.

2. PROBLEM FORMULATION

We consider a typical scenario where a microgrid orchestrates different energy generation sources to minimize cost for satisfying both local electricity and head demands simultaneously, while meeting operational constraints of electricity system. We will formulate a microgrid cost minimization problem (**MCMP**) that incorporates intermittent energy demands, time-varying electricity prices, local generation capabilities and co-generation.

We define the notations in Table 1. We denote a vector by a single symbol, *e.g.,* $a \triangleq (a(t))_{t=1}^{T}$. We also define the acronyms for our problems and algorithms in Table 2.

T	The total number of intervals (unit: min)
N	The total number of local generators
β	The startup cost of local generator ($)
c_m	The sunk cost per interval of running local generator ($)
c_o	The incremental operational cost per interval of running local generator to output an additional unit of power ($/Watt)
c_g	The price per unit of heat obtained externally using natural gas ($/Watt)
T_{on}	The minimum on-time of generator, once it is turned on
T_{off}	The minimum off-time of generator, once it is turned off
R_{up}	The maximum ramping-up rate (Watt/min)
R_{dw}	The maximum ramping-down rate (Watt/min)
L	The maximum power output of generator (Watt)
η	The heat recovery efficiency of co-generation
$a(t)$	The net power demand minus the instantaneous wind power supply and stored power from battery (Watt)
$h(t)$	The space heating demand (Watt)
$p(t)$	The spot price per unit of power obtained from the electricity grid ($P_{\min} \leq p(t) \leq P_{\max}$) ($/Watt)
$\sigma(t)$	The joint input at time t: $\sigma(t) \triangleq (a(t), h(t), p(t))$
$y_n(t)$	The on/off status of the i-th local generator (on as "1" and off as "0"), $1 \leq n \leq N$
$u_n(t)$	The power output level when the i-th generator is on (Watt), $1 \leq n \leq N$
$s(t)$	The heat level obtained externally by natural gas (Watt)
$v(t)$	The power level obtained from electricity grid (Watt)

Table 1: Notations of formulation. Brackets indicate the unit.

MCMP	Microgrid Cost Minimization Problem
fMCMP	**MCMP** for Fast-responding Case
$\mathbf{fMCMP}_{\text{s}}$	**fMCMP** with Single Generator
SP	Simplified Problem of $\mathbf{fMCMP}_{\text{s}}$
CHASE_s	The basic version of CHASE for $\mathbf{fMCMP}_{\text{s}}$
CHASE_{s+}	CHASE for $\mathbf{fMCMP}_{\text{s}}$
$\mathsf{CHASE}_s^{\text{lk}(\omega)}$	The basic version of CHASE for $\mathbf{fMCMP}_{\text{s}}$ with look-ahead
$\mathsf{CHASE}_{s+}^{\text{lk}(\omega)}$	CHASE for $\mathbf{fMCMP}_{\text{s}}$ with look-ahead
$\mathsf{CHASE}^{\text{lk}(\omega)}$	CHASE for **fMCMP** with look-ahead
$\mathsf{CHASE}_{\text{gen}}^{\text{lk}(\omega)}$	CHASE for **MCMP** with look-ahead

Table 2: Acronyms for problems and algorithms.

2.1 Model

Intermittent Energy Demands: We consider arbitrary renewable energy supply (*e.g.,* wind). Let the net demand (*i.e.,* the residual electricity demand not balanced by wind generation) at time t be $a(t)$. Note that we do not rely on any specific stochastic model of $a(t)$.

External Power from Electricity Grid: The microgrid can obtain external electricity supply from the central grid for unbalanced electricity demand in an on-demand manner. We let the spot price at time t from electricity grid be $p(t)$. We assume that $P_{\min} \le p(t) \le P_{\max}$. Again, we do not rely on any specific stochastic model on $p(t)$.

Local Generators: The microgrid has N units of homogeneous local generators, each having an maximum power output capacity L. Based on a common generator model [21], we denote β as the startup cost of turning on a generator, c_m as the sunk cost of maintaining a generator in its active state per unit time, and c_o as the operational cost per unit time for an active generator to output an additional unit of energy. Furthermore, a more realistic model of generators considers advanced *operational constraints*:

1. *Minimum On/Off Periods*: If one generator has been committed (or uncommitted) at time t, it must remain committed (uncommitted resp.) until time $t + \mathsf{T}_{\text{on}}$ ($t + \mathsf{T}_{\text{off}}$ resp.).
2. *Ramping-up/down Rates*: The incremental power output in two consecutive time intervals is limited by the ramping-up and ramping-down constraints.

Most microgrids today employ generators powered by gas turbines or diesel engines. These generators are "fast-responding" in the sense that they can be powered up in several minutes, and have small minimum on/off periods as well as large ramping-up/down rates. Meanwhile, there are also generators based on steam engine, and are "slow-responding" with non-negligible T_{on}, T_{off}, and small ramping-up/down rates.

Co-generation and Heat Demand: The local CHP generators can simultaneously generate electricity and useful heat. Let the heat recovery efficiency for co-generation be η, *i.e.,* for each unit of electricity generated, η unit of useful heat can be supplied for free. Alternatively, without co-generation, heating can be generated separately using external natural gas, which costs c_g per unit time. Thus, ηc_g is the saving due to using co-generation to supply heat, provided that there is sufficient heat demand. We assume $c_o \ge \eta \cdot c_g$. In other words, it is cheaper to generate heat by natural gas than purely by generators (if not considering the benefit of co-generation). Note that a system with no co-generation can be viewed as a special case of our model by setting $\eta = 0$. Let the demand of heat at time t be $h(t)$.

To keep the problem interesting, we assume that $c_o + \frac{c_m}{L} < P_{\max} + \eta \cdot c_g$. This ensures that the minimum co-generation energy cost is cheaper than the maximum external energy price. If this is not the case, it is always optimal to obtain power and heat externally.

2.2 Problem Definition

We divide a finite time horizon into T discrete time slots, each is assumed to have a unit length without loss of generality. The microgrid operational and generation cost in $[1, T]$ is given by

$$\text{Cost}(y,u,v,s) \triangleq \sum_{t=1}^{T} \Big\{ p(t)\cdot v(t) + c_g \cdot s(t) + \sum_{n=1}^{N} \left[c_o \cdot u_n(t) + c_m \cdot y_n(t) + \beta [y_n(t) - y_n(t-1)]^+ \right] \Big\}, \tag{1}$$

which includes the cost of grid electricity, the cost of the external gas, and the operating and switching cost of local CHP generators in the entire horizon $[1, T]$. Throughout this paper, we set initial conditions $y_n(0) = 0$, $1 \le n \le N$.

We formally define the **MCMP** as a mixed-integer programming problem, given electricity demand a, heat demand h, and grid electricity price p as time-varying inputs:

$$\min_{y,u,v,s} \text{Cost}(y,u,v,s) \tag{2a}$$
$$\text{s.t. } 0 \le u_n(t) \le L \cdot y_n(t), \tag{2b}$$
$$\textstyle\sum_{n=1}^{N} u_n(t) + v(t) \ge a(t), \tag{2c}$$
$$\eta \cdot \textstyle\sum_{n=1}^{N} u_n(t) + s(t) \ge h(t), \tag{2d}$$
$$u_n(t) - u_n(t-1) \le \mathsf{R}_{\text{up}}, \tag{2e}$$
$$u_n(t-1) - u_n(t) \le \mathsf{R}_{\text{dw}}, \tag{2f}$$
$$y_n(\tau) \ge \mathbf{1}_{\{y_n(t) > y_n(t-1)\}}, t{+}1 \le \tau \le t{+}\mathsf{T}_{\text{on}}\text{-}1, \tag{2g}$$
$$y_n(\tau) \le 1\text{-}\mathbf{1}_{\{y_n(t) < y_n(t-1)\}}, t{+}1 \le \tau \le t{+}\mathsf{T}_{\text{off}}\text{-}1, \tag{2h}$$
$$\text{var } y_n(t) \in \{0,1\}, u_n(t), v(t), s(t) \in \mathbb{R}_0^+, n \in [1,N], t \in [1,T],$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function and $\mathbb{R}_0^+$ represents the set of non-negative numbers. The constraints are similar to those in the power system literature and capture the operational constraints of generators. Specifically, constraint (2b) captures the constraint of maximal output of the local generator. Constraints (2c)-(2d) ensure that the demands of electricity and heat can be satisfied, respectively. Constraints (2e)-(2f) capture the constraints of maximum ramping-up/down rates. Constraints (2g)-(2h) capture the minimum on/off period constraints (note that they can also be expressed in linear but hard-to-interpret forms).

3. FAST-RESPONDING GENERATOR CASE

This section considers the fast-responding generator scenario. Most CHP generators employed in microgrids are based on gas or diesel. These generators can be fired up in several minutes and have high ramping-up/down rates. Thus at the timescale of energy generation (usually tens of minutes), they can be considered as having no minimum on/off periods and ramping-up/down rate constraints. That is, $\mathsf{T}_{\text{on}} = 0$, $\mathsf{T}_{\text{off}} = 0$, $\mathsf{R}_{\text{up}} = \infty$, $\mathsf{R}_{\text{dw}} = \infty$. We remark that this model captures most microgrid scenarios today. We will extend the algorithm developed for this responsive generator scenario to general generator scenario in Sec. 4.

To proceed, we first study a simple case where there is one unit of generator. We then extend the results to N units of homogenous generators in Sec. 3.3.

3.1 Single Generator Case

We first study a basic problem that considers a single generator. Thus, we can drop the subscript n (the index of the generator) when there is no source of confusion:

$$\textbf{fMCMP}_{\text{s}} : \min_{y,u,v,s} \text{Cost}(y,u,v,s) \tag{3a}$$
$$\text{s.t. } 0 \le u(t) \le L \cdot y(t), \tag{3b}$$
$$u(t) + v(t) \ge a(t), \tag{3c}$$
$$\eta \cdot u(t) + s(t) \ge h(t), \tag{3d}$$
$$\text{var } y(t) \in \{0,1\}, u(t), v(t), s(t) \in \mathbb{R}_0^+, t \in [1,T].$$

Note that even this simpler problem is challenging to solve. First, even to obtain an offline solution (assuming com-

plete knowledge of future information), we must solve a mixed-integer optimization problem. Further, the objective function values across different slots are correlated via the startup cost $\beta[y(t)-y(t-1)]^+$, and thus cannot be decomposed. Finally, to obtain an online solution we do not even know the future.

Next, we introduce the following lemma to simplify the structure of the problem. Note that if $(y(t))_{t=1}^T$ are given, the startup cost is determined. Thus, the problem in (3a)-(3d) reduces to a linear program and can be solved independently for each slot. We then obtain the following.

Lemma 1. *Given $(\sigma(t))_{t=1}^T$ and $(y(t))_{t=1}^T$, the solution $(u(t),v(t),s(t))_{t=1}^T$ that minimizes $\mathrm{Cost}(y,u,v,s)$ is given by:*

$$u(t)=\begin{cases}0, & \text{if } p(t)+\eta\cdot c_g\le c_o\\ \min\left\{\frac{h(t)}{\eta},a(t),L\cdot y(t)\right\}, & \text{if } p(t)<c_o<p(t)+\eta\cdot c_g\\ \min\left\{a(t),L\cdot y(t)\right\}, & \text{if } c_o\le p(t)\end{cases} \tag{4}$$

and

$$v(t)=[a(t)-u(t)]^+,\quad s(t)=[h(t)-\eta\cdot u(t)]^+. \tag{5}$$

The result of Lemma 1 can be interpreted as follows. If the grid price is very high (*i.e.,* higher than c_o), then it is always more economical to use local generation as much as possible, without even considering heating. However, if the grid price is between c_o and $c_o-\eta\cdot c_g$, local electricity generation alone is not economical. Rather, it is the benefit of supplying heat through co-generation that makes local generation more economical. Hence, the amount of local generation must consider the heat demand $h(t)$. Finally, when the grid price is very low (*i.e.,* lower than $c_o-\eta\cdot c_g$), it is always more cost-effective not to use local generation.

Using Lemma 1, the problem (3a)-(3d) can be simplified to the following problem **SP** with respect to a single set of variables $y(t)$:

$$\mathbf{SP}:\min_y\ \mathrm{Cost}(y)$$
$$\text{var}\ \ y(t)\in\{0,1\},t\in[1,T],$$

where

$$\mathrm{Cost}(y)\triangleq\sum_{t=1}^T\Big(\psi\big(\sigma(t),y(t)\big)+\beta\cdot[y(t)-y(t-1)]^+\Big)$$

$\psi\big(\sigma(t),y(t)\big)\triangleq c_ou(t)+p(t)v(t)+c_gs(t)+c_my(t)$ and $(u(t),v(t),s(t))$ are defined according to Lemma 1.

In other words, we only need to consider turning on ($y(t)=1$) or off ($y(t)=0$) the generator based on the trajectories of $\psi\big(\sigma(t),y(t)\big)$.

Remark: Readers familiar with the competitive online algorithms for server scheduling in data centers [24, 25] may see some similarity between the problem **SP** and those in [24, 25], *i.e.,* both deal the scheduling difficulty introduced by the startup cost β. Despite such similarity, however, both the inherent structure and the solution of our problem **SP** are significantly different. First, in the server scheduling problem in data centers, the online algorithm only needs to be concerned with the switching decision in one direction, *i.e.,* turning off idle servers when the demand is low. On the other hand, when the demand is high, additional servers must always be immediately turned on. In contrast, in our problem **SP** we must consider the switching decisions in both directions, *i.e.,* either when it is more (or less, correspondingly) economical to use local generation, one may wait until a later time to switch on (resp. off) local generation. Second, the solutions in [24, 25] essentially depend on *consecutive* service idling intervals. In contrast, as we will see soon in Sec. 3.1.1, both the offline and online solutions for our problem **SP** depend on the *cumulative* cost difference $\Delta(t)$ defined in (7). Correspondingly, the proof techniques are also significantly different.

3.1.1 Offline Optimal Solution

We first study the offline setting, where the input $(\sigma(t))_{t=1}^T$ is given ahead of time. We will reveal an elegant structure of the optimal solution. Then, in Section 3.1.2 we will exploit this structure to design an efficient online algorithm.

Define

$$\delta(t)\triangleq\psi\Big(\sigma(t),0\Big)-\psi\Big(\sigma(t),1\Big). \tag{6}$$

$\delta(t)$ can be interpreted as the one-slot cost difference between using or not using local generation. Intuitively, if $\delta(t)>0$ (resp. $\delta(t)<0$), it will be desirable to turn on (resp. off) the generator. However, due to the startup cost, we should not turn on and off the generator too frequently. Instead, we should evaluate whether the *cumulative* gain or loss in the future can offset the startup cost. This intuition motivates us to define the following cumulative cost difference $\Delta(t)$. We set the initial value as $\Delta(0)=-\beta$ and define $\Delta(t)$ inductively:

$$\Delta(t)\triangleq\min\Big\{0,\max\{-\beta,\Delta(t-1)+\delta(t)\}\Big\}. \tag{7}$$

Note that $\Delta(t)$ is only within the range $[-\beta,0]$. Otherwise, the minimum cap ($-\beta$) and maximum cap (0) will apply to retain $\Delta(t)$ within $[-\beta,0]$. An important feature of $\Delta(t)$ useful later in online algorithm design is that it can be computed given the past and current input $\sigma(\tau)$, $1\le\tau\le t$.

Next, we construct critical segments according to $\Delta(t)$, and then classify segments by types. Each type of segments captures similar episodes of demands. As shown later in Theorem 1, it suffices to solve the cost minimization problem over every segment and combine their solutions to obtain an offline optimal solution for the overall problem **SP**.

Definition 1. We divide all time intervals in $[1,T]$ into disjoint parts called *critical segments*:

$$[1,T_1^c],[T_1^c+1,T_2^c],[T_2^c+1,T_3^c],...,[T_k^c+1,T]$$

The critical segments are characterized by a set of *critical points*: $T_1^c<T_2^c<...<T_k^c$. We define each critical point T_i^c along with an auxiliary point $\tilde{T_i^c}$, such that the pair $(T_i^c,\tilde{T_i^c})$ satisfy the following conditions:

- (Boundary): Either $\big(\Delta(T_i^c)=0$ and $\Delta(\tilde{T_i^c})=-\beta\big)$ or $\big(\Delta(T_i^c)=-\beta$ and $\Delta(\tilde{T_i^c})=0\big)$.
- (Interior): $-\beta<\Delta(\tau)<0$ for all $T_i^c<\tau<\tilde{T_i^c}$.

In other words, each pair of $(T_i^c,\tilde{T_i^c})$ corresponds to an interval where $\triangle(t)$ goes from -β to 0 or 0 to -β, without reaching the two extreme values inside the interval. For example, $(T_1^c,\tilde{T_1^c})$ and $(T_2^c,\tilde{T_2^c})$ in Fig. 3 are two such pairs, while the corresponding critical segments are (T_1^c,T_2^c) and (T_2^c,T_3^c). It is straightforward to see that all $(T_i^c,\tilde{T_i^c})$ are uniquely defined, and hence critical segments are well-defined. See Fig. 3 for an example.

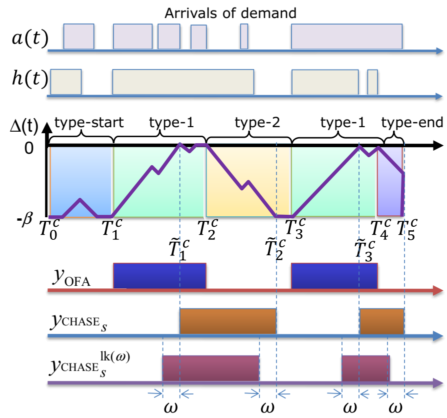


Figure 3: An example of $\Delta(t)$, y_{OFA}, y_{CHASE_s} and $y_{\text{CHASE}_s^{\text{lk}(\omega)}}$. Here, $a(t) \in \{0, 1\}$, $h(t) \in \{0, \eta\}$, and $p(t)$ is a constant in $(c_o - \eta \cdot c_g, c_o)$. We compute $\Delta(t)$ according to $a(t)$ and $h(t)$. We set the parameters so that $\Delta(t)$ increases if and only if $a(t) = 1$ and $h(t) = \eta$, to ease the discussions. The solutions y_{OFA}, y_{CHASE_s} and $y_{\text{CHASE}_s^{\text{lk}(\omega)}}$ are obtained accordingly to (8), Algorithms 1 and 3, respectively.

Once the time horizon $[1, T]$ is divided into critical segments, we can now characterize the optimal solution.

Definition 2. We classify the *type* of a critical segment by:

- *Type-start* (also call *type-0*): $[1, T_1^c]$
- *Type-1*: $[T_i^c+1, T_{i+1}^c]$, if $\Delta(T_i^c) = -\beta$ and $\Delta(T_{i+1}^c) = 0$
- *Type-2*: $[T_i^c+1, T_{i+1}^c]$, if $\Delta(T_i^c) = 0$ and $\Delta(T_{i+1}^c) = -\beta$
- *Type-end* (also call *type-3*): $[T_k^c + 1, T]$

For completeness, we also let $T_0^c = 0$ and $T_{k+1}^c = T$.

We define the cost with regard to a segment i by:

$$\text{Cost}^{\text{sg-}i}(y) \triangleq \sum_{t=T_i^c+1}^{T_{i+1}^c} \psi\big(\sigma(t), y(t)\big) + \sum_{t=T_i^c+1}^{T_{i+1}^c+1} \beta \cdot [y(t) - y(t-1)]^+$$

and define a subproblem for critical segment i by:

$$\begin{aligned} \mathbf{SP}^{\text{sg-}i}(\mathrm{y}_i^l, \mathrm{y}_i^r) : \min\ & \text{Cost}^{\text{sg-}i}(y) \\ \text{s.t. } & y(T_i^c) = \mathrm{y}_i^l,\ y(T_{i+1}^c + 1) = \mathrm{y}_i^r, \\ \text{var } & y(t) \in \{0, 1\}, t \in [T_i^c + 1, T_{i+1}^c]. \end{aligned}$$

Note that due to the startup cost across segment boundaries, in general $\text{Cost}(y) \neq \sum \text{Cost}^{\text{sg-}i}(y)$. In other words, we should not expect that putting together the solutions to each segment will lead to an overall optimal offline solution. However, the following lemma shows an important structure property that one optimal solution of $\mathbf{SP}^{\mathbf{sg-i}}(y_i^l, y_i^r)$ is independent of boundary conditions (y_i^l, y_i^r) although the optimal value depends on boundary conditions.

LEMMA 2. $(y_{\text{OFA}}(t))_{t=T_i^c+1}^{T_{i+1}^c}$ *in* (8) *is an optimal solution for* $\mathbf{SP}^{\text{sg-}i}(\mathrm{y}_i^l, \mathrm{y}_i^r)$*, despite any boundary conditions* $(\mathrm{y}_i^l, \mathrm{y}_i^r)$*.*

This lemma can be intuitively explained by Fig. 3. In type-1 critical segment, $\Delta(t)$ has an increment of β, which means that setting $y(t) = 1$ over the entire segment provides at least a benefit of β, as compared to keeping $y(t) = 0$. Such benefit compensates the possible startup cost β if the boundary conditions are not aligned with $y(t) = 1$. Therefore, regardless of the boundary conditions, we should set $y(t) = 1$ on type-1 critical segment. Other types of critical segments can be explained similarly.

We then use this lemma to show the following main result on the structure of the offline optimal solution.

Theorem 1. An optimal solution for **SP** is given by

$$y_{\text{OFA}}(t) \triangleq \begin{cases} 0, & \text{if } t \in [T_i^c + 1, T_{i+1}^c] \text{ is type-start/-2/-end}, \\ 1, & \text{if } t \in [T_i^c + 1, T_{i+1}^c] \text{ is type-1}, \end{cases} \tag{8}$$

PROOF. Refer to Appendix A. □

Theorem 1 can be interpreted as follows. Consider for example a type-1 critical segment in Fig. 3 that starts from T_1^c. Since the value of $\Delta(t)$ increases from $-\beta$ after T_1^c, it implies that $\delta(t) > 0$, and thus we are interested in turning on the generator. The difficulty, however, is that immediately after T_1^c we do not know whether the future gain by turning on the generator will offset the startup cost. On the other hand, once $\Delta(t)$ reaches 0, it means that the cumulative gain in the interval $[T_1^c, \tilde{T}_1^c]$ will be no less than the startup cost. Hence, we can safely turn on the generator at T_1^c. Similarly, for each type-2 segment we can turn off the generator at the beginning of the segment. (We note that our offline solution turns on/off the generator at the beginning of each segment because all future information is assumed to be known.)

The optimal solution is easy to compute. More importantly, the insights help us design the online algorithms.

3.1.2 *Our Proposed Online Algorithm* CHASE

Denote an online algorithm for problem **SP** by $\mathcal{A}$. We define the competitive ratio of $\mathcal{A}$ by:

$$\mathsf{CR}(\mathcal{A}) \triangleq \max_{\sigma} \frac{\text{Cost}(y_{\mathcal{A}})}{\text{Cost}(y_{\text{OFA}})} \tag{9}$$

Recall the structure of optimal solution y_{OFA}: once the process is entering type-1 (or correspondingly type-2) critical segment, we should set $y(t) = 1$ (or correspondingly $y(t) = 0$). However, the difficulty lies in determining the beginnings of type-1 and type-2 critical segments without future information. Fortunately, as illustrated in Fig. 3, it is certain that the process is in a type-1 critical segment when $\Delta(t)$ reaches 0 for the first time after hitting $-\beta$. This observation motivates us to use the algorithm CHASE_s, which is given in Algorithm 1. If $-\beta < \Delta(t) < 0$, CHASE_s maintains $y(t) = y(t-1)$ (since we do not know whether a new segment has started yet.) However, when $\Delta = 0$ (resp. $\Delta(t) = -\beta$), we know for sure that we are inside a new type-1 (resp. type-2) segment. Hence, CHASE_s sets $y(t) = 1$ (resp. $y(t) = 0$). Intuitively, the behavior of CHASE_s is to track the offline optimal in an online manner: we change the decision only after we are certain that the offline optimal decision is changed.

Even though CHASE_s is a simple algorithm, it has a strong performance guarantee, as given by the following theorem.

Theorem 2. The competitive ratio of CHASE_s satisfies

$$\mathsf{CR}(\mathsf{CHASE}_s) \leq 3 - 2\alpha < 3, \tag{10}$$

where

$$\alpha \triangleq (c_o + c_m/L)/(P_{\max} + \eta \cdot c_g) \in (0, 1] \tag{11}$$

captures the maximum price discrepancy between using local generation and external sources to supply energy.

Algorithm 1 $\mathsf{CHASE}_s[t, \sigma(t), y(t-1)]$

Find $\Delta(t)$
if $\Delta(t) = -\beta$ **then**
  $y(t) \leftarrow 0$
else if $\Delta(t) = 0$ **then**
  $y(t) \leftarrow 1$
else
  $y(t) \leftarrow y(t-1)$
end if
set $u(t)$, $v(t)$, and $s(t)$ according to (4) and (5)
Return $(y(t), u(t), v(t), s(t))$

Proof. Refer to Appendix B. □

Remark: (i) The intuition that CHASE_s is competitive can be explained by studying its worst case input shown in Fig. 4. The demands and prices are chosen in a way such that in interval $[T_0^c, T_1^c]$ $\Delta(t)$ increases from $-\beta$ to 0, and in interval $[T_1^c, T_2^c]$ $\Delta(t)$ decreases from 0 to $-\beta$. We see that in the worst case, y_{CHASE_s} never matches y_{OFA}. But even in this worst case, CHASE_s pays only 2β more than the offline solution y_{OFA} on $[T_0^c, T_2^c]$, while y_{OFA} pays at least a startup cost of of β at time T_0^c. Hence, the ratio of the online cost over the offline cost cannot be too bad. (ii) Theorem 2 says that CHASE_s is more competitive when α is large than it is small. This can be explained intuitively as follows. Large α implies small economic advantage of using local generation over external sources to supply energy. Consequently, the offline solution tends to use local generation less. It turns out CHASE_s will also use less local generation[2] and is competitive to offline solution. Meanwhile, when α is small, CHASE_s starts to use local generation. However, using local generation incurs high risk since we have to pay the startup cost to turn on the generator without knowing whether there are sufficient demands to serve in the future. Lacking future knowledge leads to a large performance discrepancy between CHASE_s and the offline optimal solution, making CHASE_s less competitive.

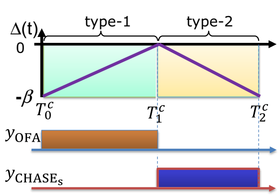


Figure 4: The worst case input of CHASE_s, and the corresponding $y_{\mathsf{CHASE_s}}$ and the offline optimal solution y_{OFA}.

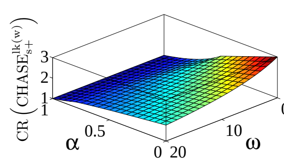


Figure 5: The competitive ratio of $\mathsf{CHASE}_{s+}^{\mathrm{lk}(\omega)}$ as a function of α and ω.

The results in Theorem 2 is strong in the sense that $\mathsf{CR}(\mathsf{CHASE}_s)$ is always upper-bounded by a small constant 3, regardless of system parameters. This is contrast to large parameter-dependent competitive ratios that one can achieve by using generic approach, *e.g.,* the metrical task system framework [8], to design online algorithms. Furthermore, we show that CHASE_s achieves close to the best possible competitive ratio for deterministic algorithms as follow.

[2] CHASE_s will turn on the local generator when $\Delta(t)$ increases to 0. The larger the α is, the slower $\Delta(t)$ increases, and less likely CHASE_s will use the local generator.

Theorem 3. Let $\epsilon > 0$ be the slot length under the discrete-time setting we consider in this paper. The competitive ratio for any deterministic online algorithm $\mathcal{A}$ for **SP** is lower bounded by

$$\mathsf{CR}(\mathcal{A}) \geq \min(3 - 2\alpha - o(\epsilon), 1/\alpha), \qquad (12)$$

where $o(\epsilon)$ vanishes to zero as ϵ goes to zero and the discrete-time setting approaches the continuous time setting.

Proof. Refer to Appendix C. □

Note that there is still a gap between the competitive ratios in (10) and (12). The difference is due to the term $1/\alpha = (P_{\max} + \eta \cdot c_g)/(c_o + c_m/L)$. This term can be interpreted as the competitive ratio of a naive strategy that always uses external power supply and separate heat supply. Intuitively, if this $1/\alpha$ term is smaller than $3 - 2\alpha$, we should simply use this naive strategy. This observation motivates us to develop an improved version of CHASE_s, called CHASE_{s+}, which is presented in Algorithm 2. Corollary 1 shows that CHASE_{s+} closes the above gap and achieves the asymptotic optimal competitive ratio. Note that whether or not the $1/\alpha$ term is smaller can be completely determined by the system parameters.

Corollary 1. CHASE_{s+} achieves the asymptotic optimal competitive ratio of any deterministic online algorithm, as

$$\mathsf{CR}(\mathsf{CHASE}_{s+}) \leq \min(3 - 2\alpha, 1/\alpha). \qquad (13)$$

Algorithm 2 $\mathsf{CHASE}_{s+}[t, \sigma(t), y(t-1)]$

if $1/\alpha \leq 3 - 2\alpha$ **then**
  $y(t) \leftarrow 0, \quad u(t) \leftarrow 0, \quad v(t) \leftarrow a(t), \quad s(t) \leftarrow h(t)$
  Return $(y(t), u(t), v(t), s(t))$
else
  Return $\mathsf{CHASE}_s[t, \sigma(t), y(t-1)]$
end if

3.2 Look-ahead Setting

We consider the setting where the online algorithm can predict a small window ω of the immediate future. Note that $\omega = 0$ returns to the case treated in Section 3.1.2, when there is no future information at all. Consider again a type-1 segment $[T_1^c, T_2^c]$ in Fig. 3. Recall that, when there is no future information, the CHASE_s algorithm will wait until $\tilde{T}_1^c$, *i.e.,* when $\Delta(t)$ reaches 0, to be certain that the offline solution must turn on the generator. Hence, the CHASE_s algorithm will not turn on the generator until this time. Now assume that the online algorithm has the information about the immediate future in a time window of length ω. By the time $\tilde{T}_1^c - w$, the online algorithm has already known that $\Delta(t)$ will reach 0 at time $\tilde{T}_1^c$. Hence, the online algorithm can safely turn on the generator at time $\tilde{T}_1^c - w$. As a result, the corresponding loss of performance compared to the offline optimal solution is also reduced. Specifically, even for the worst-case input in Fig. 4, there will be some overlap (of length ω) between y_{CHASE_s} and y_{OFA} in each segment. Hence, the competitive ratio should also improve with future information. This idea leads to the online algorithm $\mathsf{CHASE}_s^{\mathrm{lk}(\omega)}$, which is presented in Algorithm 3.

We can show the following improved competitive ratio when limited future information is available.

Algorithm 3 $\mathsf{CHASE}_s^{\mathrm{lk}(\omega)}[t, (\sigma(\tau))_{\tau=t}^{t+w}, y(t-1)]$

Find $(\Delta(\tau))_{\tau=t}^{t+w}$
Set $\tau' \leftarrow \min\{\tau = t, ..., t+w \mid \Delta(\tau) = 0 \text{ or } = -\beta\}$
if $\Delta(\tau') = -\beta$ **then**
  $y(t) \leftarrow 0$
else if $\Delta(\tau') = 0$ **then**
  $y(t) \leftarrow 1$
else
  $y(t) \leftarrow y(t-1)$
end if
set $u(t)$, $v(t)$, and $s(t)$ according to (4) and (5)
Return $(y(t), u(t), v(t), s(t))$

Theorem 4. The competitive ratio of $\mathsf{CHASE}_s^{\mathrm{lk}(\omega)}$ satisfies

$$\mathsf{CR}\left(\mathsf{CHASE}_s^{\mathrm{lk}(\omega)}\right) \leq 3 - 2 \cdot g(\alpha, \omega), \qquad (14)$$

where $\omega \geq 0$ is the look-ahead window size, $\alpha \in (0, 1]$ is defined in (11), and

$$g(\alpha, \omega) = \alpha + \frac{(1-\alpha)}{1 + \beta\left(Lc_o + c_m/(1-\alpha)\right)/\left(\omega(Lc_o + c_m)c_m\right)}. \qquad (15)$$

captures the benefit of looking-ahead and monotonically increases from α to 1 as ω increases. In particular, $\mathsf{CR}(\mathsf{CHASE}_s^{\mathrm{lk}(0)}) = \mathsf{CR}(\mathsf{CHASE}_s)$.

Proof. Refer to Appendix D. □

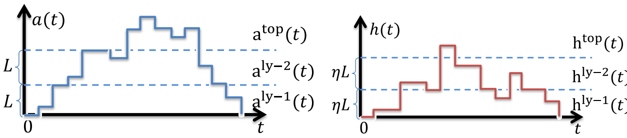
We replace CHASE_s by $\mathsf{CHASE}_s^{\mathrm{lk}(\omega)}$ in CHASE_{s+} and obtain an improved algorithm for the look-ahead setting, named $\mathsf{CHASE}_{s+}^{\mathrm{lk}(\omega)}$. Fig. 5 shows the competitive ratio of $\mathsf{CHASE}_{s+}^{\mathrm{lk}(\omega)}$ in (14) as a function of α and ω.

3.3 Multiple Generator Case

Now we consider the general case with N units of homogeneous generators, each having an maximum power capacity L, startup cost β, sunk cost c_m and per unit operational cost c_o. We define a generalized version of problem:

$$\begin{aligned} \textbf{fMCMP}: \min_{y,u,v,s} & \ \mathrm{Cost}(y, u, v, s) \\ \text{s.t. } & \text{Constraints (2b), (2c), and (2d)} \\ \text{var } & y_n(t) \in \{0, 1\}, u_n(t), v(t), s(t) \in \mathbb{R}_0^+, \\ & n \in [1, N], t \in [1, T]. \end{aligned}$$

Next, we will construct both offline and online solutions to **fMCMP** in a divide-and-conquer fashion. We will first partition the demands into sub-demands for each generator, and then optimize the local generation *separately* for each sub-demand. Note that the key is to correctly partition the demand so that the combined solution is still optimal. Our strategy below essentially slices the demand (as a function of t) into multiple layers from the bottom up (see Fig. 6). Each layer has at most L units of electricity demand and $\eta \cdot L$ units of heat demand. The intuition here is that the layers at the bottom exhibit the least frequent variations of demand. Hence, by assigning each of the layers at the bottom to a dedicated generator, these generators will incur the least amount of switching, which helps to reduce the start-up cost.



(a) An example of $(a^{\mathrm{ly-n}})$. (b) An example of $(h^{\mathrm{ly-n}})$.

Figure 6: An example of $(a^{\mathrm{ly-n}})$ and $(h^{\mathrm{ly-n}})$. In this example, $N = 2$. We obtain 3 layers of electricity and heat demands, respectively.

More specifically, given $(a(t), h(t))$, we slice them into $N+1$ layers:

$$a^{\text{ly-1}}(t) = \min\{L, a(t)\}, \quad h^{\text{ly-1}}(t) = \min\{\eta \cdot L, h(t)\} \quad (16a)$$
$$a^{\text{ly-}n}(t) = \min\{L, a(t)\text{-}\textstyle\sum_{r=1}^{n-1} a^{\text{ly-}r}(t)\}, n \in [2, N] \quad (16b)$$
$$h^{\text{ly-}n}(t) = \min\{\eta \cdot L, h(t)\text{-}\textstyle\sum_{r=1}^{n-1} h^{\text{ly-}r}(t)\}, n \in [2, N] \quad (16c)$$
$$a^{\text{top}}(t) = \min\{L, a(t) - \textstyle\sum_{r=1}^{N} a^{\text{ly-}r}(t)\} \quad (16d)$$
$$h^{\text{top}}(t) = \min\{\eta \cdot L, h(t) - \textstyle\sum_{r=1}^{N} h^{\text{ly-}r}(t)\} \quad (16e)$$

It is easy to see that electricity demand satisfies $a^{\text{ly-}n}(t) \leq L$ and heat demand satisfies $h^{\text{ly-}n}(t) \leq \eta \cdot L$. Thus, each layer of sub-demand can be served by a single local generator if needed. Note that $(a^{\text{top}}, h^{\text{top}})$ can only be satisfied from external supplies, because they exceed the capacity of local generation.

Based on this decomposition of demand, we then decompose the **fMCMP** problem into N sub-problems $\textbf{fMCMP}_{\mathrm{s}}^{\text{ly-}n}$ $(1 \leq n \leq N)$, each of which is an $\textbf{fMCMP}_{\mathrm{s}}$ problem with input $(a^{\text{ly-}n}, h^{\text{ly-}n}, p)$. We then apply the offline and online algorithms developed earlier to solve each sub-problem $\textbf{fMCMP}_{\mathrm{s}}^{\text{ly-}n}$ $(1 \leq n \leq N)$ *separately.* By combining the solutions to these sub-problems, we obtain offline and online solutions to **fMCMP**. For the offline solution, the following theorem states that such a divide-and-conquer approach results in no optimality loss.

Theorem 5. Suppose (y_n, u_n, v_n, s_n) is an optimal offline solution for each $\textbf{fMCMP}_{\mathrm{s}}^{\text{ly-}n}$ $(1 \leq n \leq N)$. Then $((y_n^*, u_n^*)_{n=1}^N, v^*, s^*)$ defined as follows is an optimal offline solution for **fMCMP**:

$$\begin{aligned} y_n^*(t) &= y_n(t), \quad v^*(t) = a^{\text{top}}(t) + \textstyle\sum_{n=1}^N v_n(t) \\ u_n^*(t) &= u_n(t), \quad s^*(t) = h^{\text{top}}(t) + \textstyle\sum_{n=1}^N s_n(t) \end{aligned} \qquad (17)$$

Proof. Refer to Appendix E. □

For the online solution, we also apply such a divide-and-conquer approach by using (i) a central demand dispatching module that slices and dispatches demands to individual generators according to (16a)-(16e), and (ii) an online generation scheduling module sitting on each generator n $(1 \leq n \leq N)$ *independently* solving their own $\textbf{fMCMP}_s^{\text{ly-}n}$ sub-problem using the online algorithm $\mathsf{CHASE}_{s+}^{\mathrm{lk}(\omega)}$.

The overall online algorithm, named $\mathsf{CHASE}^{\mathrm{lk}(\omega)}$, is simple to implement without the need to coordinate the control among multiple local generators. Since the offline (resp. online) cost of **fMCMP** is the sum of the offline (resp. online) costs of $\textbf{fMCMP}_s^{\text{ly-}n}$ $(1 \leq n \leq N)$, it is not difficult to establish the competitive ratio of $\mathsf{CHASE}^{\mathrm{lk}(\omega)}$ as follows.

Theorem 6. The competitive ratio of $\mathsf{CHASE}^{\mathrm{lk}(\omega)}$ satisfies

$$\mathsf{CR}(\mathsf{CHASE}^{\mathrm{lk}(\omega)}) \leq \min(3 - 2 \cdot g(\alpha, \omega), 1/\alpha), \tag{18}$$

where $\alpha \in (0, 1]$ is defined in (11) and $g(\alpha, \omega) \in [\alpha, 1]$ is defined in (15).

4. SLOW-RESPONDING GENERATOR CASE

We next consider the slow-responding generator case, with the generators having non-negligible constraints on the minimum on/off periods and the ramp-up/down speed. For this slow-responding version of **MCMP**, its offline optimal solution is harder to characterize than **fMCMP** due to the additional challenges introduced by the cross-slot constraints (2e)-(2h). Following the standard back-tracking arguments, we have designed an offline algorithm that takes the complete input σ and outputs an offline optimal solution. However, it does not provide insights for designing online algorithms and we skip it here due to the space limitation. Instead, we extend our $\mathsf{CHASE}^{\mathrm{lk}(\omega)}$ algorithm developed early to the slow-responding scenario and provide performance guarantee.

In the slow-responding setting, local generators cannot be turned on and off immediately when demand changes. Rather, if a generator is turned on at time t, it must remain on for at least T_{on} time, during which we say that the generator is *unavailable for stoppage*. Similarly, if the generator is turned off at time t, it must remain off for at least $\mathsf{T}_{\mathrm{off}}$ time, during which we say that the generator is *unavailable for service*. Further, the changes of $u_n(t) - u_n(t-1)$ must be bounded by R_{up} and $-\mathsf{R}_{\mathrm{down}}$.

A simple heuristic is to base the decisions first on $\mathsf{CHASE}^{\mathrm{lk}(\omega)}$, and then modify the decisions to respect the above constraints. More specifically, one can obtain energy supply from external sources when local generators are *unavailable for service*, and keep local generators on for a minimum T_{on} period when they are *unavailable for stoppage*. Further, the value of $u_n(t)$ can be adjusted based on the ramp-up/down constraints. We name this heuristic by $\mathsf{CHASE}_{\mathrm{gen}}^{\mathrm{lk}(\omega)}$. For simplicity we present its single-generator version in Algorithm 4, which can be easily extended to multiple generator scenarios following the divide-and-conquer approach elaborated in Sec. 3.3. We derive an upper bound on the competitive ratio of $\mathsf{CHASE}_{\mathrm{gen}}^{\mathrm{lk}(\omega)}$ as follows.

Theorem 7. The competitive ratio of $\mathsf{CHASE}_{\mathrm{gen}}^{\mathrm{lk}(\omega)}$ is upper bounded by $(3 - 2g(\alpha, \omega)) \cdot \max(r_1, r_2)$, where $g(\alpha, \omega)$ is defined in (15) and

$$r_1 = 1 + \max\left\{ \frac{(P_{\max} + c_g \cdot \eta - c_0)}{Lc_0 + c_m} \max\{0, (L - \mathsf{R}_{\mathrm{up}})\} \right.$$
$$\left. \frac{c_o}{c_m} \max\{0, (L - \mathsf{R}_{\mathrm{dw}})\} \right\},$$

and $r_2 = \dfrac{\beta + c_m \cdot \mathsf{T}_{\mathrm{on}}}{\beta} + \dfrac{L(P_{\max} + c_g \cdot \eta)}{\beta} (\mathsf{T}_{\mathrm{on}} + \mathsf{T}_{\mathrm{off}})$.

PROOF. Refer to Appendix F. □

We note that when $\mathsf{T}_{\mathrm{on}} = \mathsf{T}_{\mathrm{off}} = 0$, $\mathsf{R}_{\mathrm{up}} = \mathsf{R}_{\mathrm{dw}} = \infty$, the above upper bound matches that of $\mathsf{CHASE}^{\mathrm{lk}(\omega)}$ in Theorem 6 (specifically the first term inside the min function).

Algorithm 4 $\mathsf{CHASE}_{\mathrm{gen}}^{\mathrm{lk}(\omega)}[t, (\sigma(\tau))_{\tau=1}^{t+\omega}, y(t-1)]$

$(y_s(t), u_s(t), v_s(t), s_s(t)) \leftarrow \mathsf{CHASE}_s^{\mathrm{lk}(\omega)}[t, (\sigma(\tau))_{\tau=1}^{t+w}, y(t\text{-}1)]$
if $y_s(t) = 1$ **then**
  if $y(t') = 0,\ \forall t' \in [\max(1, t - \mathsf{T}_{\mathrm{on}}), t-1]$, **then**
    $y(t) \leftarrow y_s(t)$
  else
    $y(t) \leftarrow y(t-1)$
  end if
else
  if $y(t') = 1,\ \forall t' \in [\max(1, t - \mathsf{T}_{\mathrm{off}}), t-1]$, **then**
    $y(t) \leftarrow y_s(t)$
  else
    $y(t) \leftarrow y(t-1)$
  end if
end if
if $u_s(t) > u(t-1)$ **then**
  $u(t) \leftarrow u(t-1) + \min(\mathsf{R}_{\mathrm{up}}, u_s(t) - u(t-1))$
else
  $u(t) \leftarrow u(t-1) - \min(\mathsf{R}_{\mathrm{dw}}, u(t-1) - u_s(t))$
end if
$v(t) \leftarrow [a(t) - u(t)]^+$
$s(t) \leftarrow [h(t) - \eta \cdot u(t)]^+$
Return $(y_s(t), u_s(t), v_s(t), s_s(t))$

5. EMPIRICAL EVALUATIONS

We evaluate the performance of our algorithms based on evaluations using real-world traces. Our objectives are threefold: (i) evaluating the potential benefits of CHP and the ability of our algorithms to unleash such potential, (ii) corroborating the empirical performance of our online algorithms under various realistic settings, and (iii) understanding how much local generation to invest to achieve substantial economic benefit.

5.1 Parameters and Settings

Demand Trace: We obtain the demand traces from California Commercial End-Use Survey (CEUS) [1]. We focus on a college in San Francisco, which consumes about 154 GWh electricity and 5.1×10^6 therms gas per year. The traces contain hourly electricity and thermal demands of the college for year 2002. The heat demands for a typical week in summer and spring are shown in Fig. 7. They display regular daily patterns in peak and off-peak hours, and typical weekday and weekend variations.

Wind Power Trace: We obtain the wind power traces from [4]. We employ power output data for the typical weeks in summer and spring with a resolution of 1 hour of an offshore wind farm right outside San Francisco with an installed capacity of 12MW. The net electricity demand, which is computed by subtracting the wind generation from electricity demand is shown in Fig. 7. The highly fluctuating and unpredictable nature of wind generation makes it difficult for the conventional prediction-based energy generation scheduling solutions to work effectively.

Electricity and Natural Gas Prices: The electricity and natural gas price data are from PG&E [5] and are shown in table 3. Besides, the grid electricity prices for a typical week in summer and winter are shown in Fig. 7. Both the electricity demand and the price show strong diurnal properties: in the daytime, the demand and price are relatively high. At nights, both are low. This suggests the feasibility of reducing the microgrid operating cost by generating cheaper

energy locally to serve the demand during the daytime when both the demand and electricity price are high.

Generator Model: We adopt generators with specifications the same as the one in [6]. The full output of a single generator is $L = 3MW$. The incremental cost per unit time to generate an additional unit of energy c_o is set to be $0.051/KWh$, which is calculated according to the natural gas price and the generator efficiency. We set the heat to electricity ratio η to be 1.8 according to [6]. We also set the unit-time generator running cost to be $c_m = 110\$/h$, which includes the amortized capital cost and maintenance cost according to a similar setting from [33]. We set the startup cost β equivalent to running the generator at its full capacity for about 5 hrs at its own operating cost, which gives $\beta = 1400\$$. In addition, we assume for each generator $\mathsf{T}_{\text{on}} = \mathsf{T}_{\text{off}} = 3h$ and $\mathsf{R}_{\text{up}} = \mathsf{R}_{\text{dw}} = 1MW/h$, unless mentioned otherwise. For electricity demand trace we use, the peak demand is 30MW. Thus, we assume there are 10 such CHP generators so as to fully satisfy the demand.

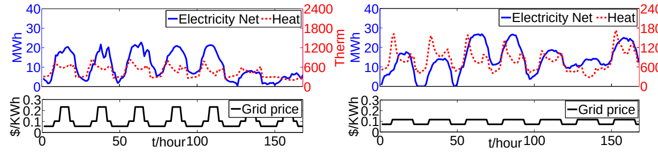
Local Heating System: We assume an on-demand heating system with capacity sufficiently large to satisfy all the heat demand by itself and without on-off cost or ramp limit. The efficiency of a heating system is set to 0.8 according to [2], and consequently we can compute the unit heat generation cost to be $c_g = 0.0179\$/KWh$.

Cost Benchmark: We use the cost incurred by using only external electricity, heating and wind energy (without CHP generators) as a benchmark. We evaluate the cost reduction due to our algorithms.

Comparisons of Algorithms: We compare three algorithms in our simulations. (1) our online algorithm CHASE; (2) the Receding Horizon Control (RHC) algorithm; and (3) the OFFLINE optimal algorithm we introduce in Sec. 4. RHC is a heuristic algorithm commonly used in the control literature [22]. In RHC, an estimate of the near future (*e.g.,* in a window of length w) is used to compute a tentative control trajectory that minimizes the cost over this time-window. However, only the first step of this trajectory is implemented. In the next time slot, the window of future estimates shifts forward by 1. Then, another control trajectory is computed based on the new future information, and again only the first step is implemented. This process then continues. We note that because at each step RHC does not consider any adversarial future dynamics beyond the time-window w, there is no guarantee that RHC is competitive. For the OFFLINE algorithm, the inputs are system parameters (such as β, c_m and T_{on}), electricity demand, heat demand, wind power output, gas price, and grid electricity price. For online algorithms CHASE and RHC, the input is the same as the OFFLINE except that at time t, only the demands, wind power output, and prices in the past and the look-ahead window (*i.e.,* $[1, t+w]$) are available. The output for all three algorithms is the total cost incurred during the time horizon $[1, T]$.

5.2 Potential Benefits of CHP

Purpose: The experiments in this subsection aim to answer two questions. First, what is the potential savings with microgrids? Note that electricity, heat demand, wind station output as well as energy price all exhibit seasonal patterns. As we can see from Figs. 7a and 7b, during summer (similarly autumn) the electricity price is high, while during winter (similarly spring) the heat demand is high. It is then



(a) Summer (b) Winter

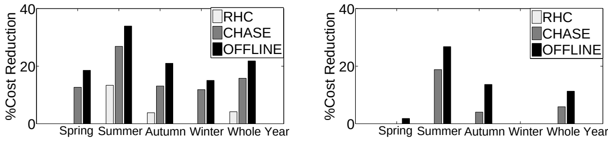
Figure 7: Electricity net demand and heat demand for a typical week in summer and winter. The net demand is computed by subtracting the wind generation from the electricity demand. The net electricity demand and the heat demand need to be satisfied by using the local CHP generators, the electricity grid, and the heating system.

Electricity	Summer (May-Oct.) $/kWh	Winter (Nov.-Apr.) $/kWh
On-peak	0.232	N/A
Mid-peak	0.103	0.116
Off-peak	0.056	0.072
Natural Gas	0.419$/therm	0.486$/therm

Table 3: PG&E commercial tariffs and natural gas tariffs. In the table, summer on-peak, mid-peak, and off-peak hours are weekday 12-18, weekday 8-12, and the remaining hours, respectively. Winter mid-peak and off-peak hours are weekday 8-22 and the remaining hours, respectively. The gas price is an average; monthly prices vary slightly according to PG&E.

interesting to evaluate under what settings and inputs the savings will be higher. Second, what is the difference in cost-savings with and without the co-generation capability? In particular, we conduct two sets of experiments to evaluate the cost reductions of various algorithms. Both experiments have the same default settings, except that the first set of experiments (referred to as CHP) assumes the CHP technology in the generators are enabled, and the second set of experiments (referred to as NOCHP) assumes the CHP technology is not available, in which case the heat demand must be satisfied solely by the heating system. In all experiments, the look-ahead window size is set to be $w = 3$ hours according to power system operation and wind generation forecast practice [3]. The cost reductions of different algorithms are shown in Fig. 8a and 8b. The vertical axis is the cost reduction as compared to the cost benchmark presented in Sec. 5.1.

Observations: First, the whole-year cost reductions obtained by OFFLINE are 21.8% and 11.3% for CHP and NOCHP scenarios, respectively. This justifies the economic potential of using local generation, especially when CHP technology is enabled. Then, looking at the seasonal performance of OFFLINE, we observe that OFFLINE achieves much more cost savings during summer and autumn than during spring and winter. This is because the electricity price during summer and autumn is very high, thus we can benefit much more from using the relatively-cheaper local generation as compared to using grid energy only. Moreover, OFFLINE achieves much more cost savings when CHP is enabled than when it is not during spring and winter. This is because, during spring and winter, the electricity price is relatively low and the heat demand is high. Hence, just using local



(a) Local generators with CHP (b) Local generators without CHP

Figure 8: Cost reductions for different seasons and the whole year.

generation to supply electricity is not economical. Rather, local generation becomes more economical only if it can be used to supply both electricity and heat together (*i.e.,* with CHP technology).

Second, CHASE performs consistently close to OFFLINE across inputs from different seasons, even though the different settings have very different characteristics of demand and supply. In contrast, the performance of RHC depends heavily on the input characteristics. For example, RHC achieves some cost reduction during summer and autumn when CHP is enabled, but achieves 0 cost reduction in all the other cases.

Ramifications: In summary, our experiments suggest that exploiting local generation can save more cost when the electricity price is high, and CHP technology is more critical for cost reduction when heat demand is high. Regardless of the problem setting, it is important to adopt an intelligent online algorithm (like CHASE) to schedule energy generation, in order to realize the full benefit of microgrids.

5.3 Benefits of Looking-Ahead

Purpose: We compare the performances of CHASE to RHC and OFFLINE for different sizes of the look-ahead window and show the results in Fig. 9. The vertical axis is the cost reduction as compared to the cost benchmark in Sec. 5.1 and the horizontal axis is the size of lookahead window, which varies from 0 to 20 hours.

Observations: We observe that the performance of our online algorithm CHASE is already close to OFFLINE even when no or little look-ahead information is available (*e.g.,* $w = 0$, 1, and 2). In contrast, RHC performs poorly when the look-ahead window is small. When w is large, both CHASE and RHC perform very well and their performance are close to OFFLINE when the look-ahead window w is larger than 15 hours.

An interesting observation is that it is more important to perform intelligent energy generation scheduling when there are no or little look-ahead information available. When there are abundant look-ahead information available, both CHASE and RHC achieve good performance and it is less critical to carry out sophisticated algorithm design.

In Fig. 11a and 11b, we separately evaluate the benefit of looking-ahead under the fast-responding and slow-responding scenarios. We evaluate the empirical competitive ratio between the cost of CHASE and OFFLINE, and compare it with the theoretical competitive ratio according to our analytical results. In the fast-responding scenario (Fig. 11a), for each generator there are no minimum on/off period and ramping-up/down constraints. Namely, $\mathsf{T}_{\text{on}} = 0$, $\mathsf{T}_{\text{off}} = 0$, $\mathsf{R}_{\text{up}} = \infty$, $\mathsf{R}_{\text{dw}} = \infty$. In the slow-responding scenario (Fig. 11b), we set $\mathsf{T}_{\text{on}} = \mathsf{T}_{\text{off}} = 3h$ and $\mathsf{R}_{\text{up}} = \mathsf{R}_{\text{dw}} = 1MW/h$. In both experiments, we observe that the theoretical ratio decreases rapidly as look-ahead window size increases. Further, the empirical ratio is already close to one even when there is no look-ahead information.

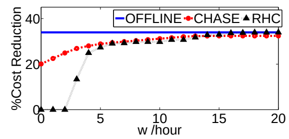


Figure 9: Cost reduction as a function of look ahead window size ω.

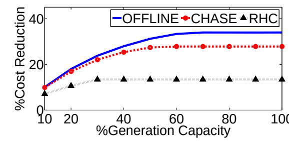
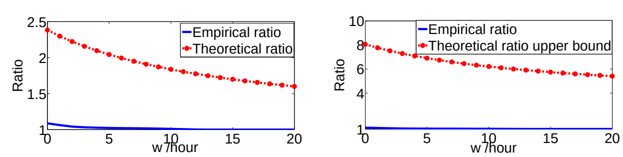


Figure 10: Cost reduction as a function of local generation capacity.

5.4 Impacts of Look-ahead Error

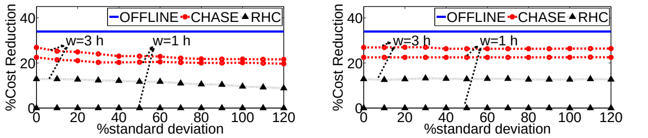
Purpose: Previous experiments show that our algorithms have better performance if a larger time-window of accurate look-ahead input information is available. The input information in the look-ahead window include the wind station power output, the electricity and heat demand, and the central grid electricity price. In practice, these look-ahead information can be obtained by applying sophisticated prediction techniques based on the historical data. However, there are always prediction errors. For example, while the day-ahead electricity demand can be predicted within 2-3% range, the wind power prediction in the next hours usually comes with an error range of 20-50% [20]. Therefore, it is important to evaluate the performance of the algorithms in the presence of prediction error.

Observations: To achieve this goal, we evaluate CHASE with look-ahead window size of 1 and 3 hours. According to [20], the hour-level wind-power prediction-error in terms of the percentage of the total installed capacity usually follows Gaussian distribution. Thus, in the look-ahead window, a zero-mean Gaussian prediction error is added to the amount of wind power in each time-slot. We vary the standard deviation of the Gaussian prediction error from 0 to 120% of the total installed capacity. Similarly, a zero-mean Gaussian prediction error is added to the heat demand, and its standard deviation also varies from 0 to 120% of the peak demand. We note that in practice, prediction errors are often in the range of 20-50% for 3-hour prediction [20]. Thus, by using a standard deviation up to 120%, we are essentially stress-testing our proposed algorithms. We average 20 runs for each algorithm and show the results in Figs. 12a and



(a) Fast-responding scenario (b) Slow-responding scenario

Figure 11: Theoretical and empirical ratios for CHASE, as functions of look-ahead window size ω. Note that the theoretical competitive ratios (or their bounds) measure the worst-case performance and are often much larger than the empirical ratios observed in practice.



(a) Wind power prediction error (b) Heat demand prediction error

Figure 12: Cost reduction as a function of the size prediction error (measured by the standard deviation of the prediction error as a percentage of (a)installed capacity and (b)peak heat demand).

12b. As we can see, both CHASE and RHC are fairly robust to the prediction error and both are more sensitive to the wind-power prediction error than to the heat-demand prediction error. Besides, the impact of the prediction error is relatively small when the look-ahead window size is small, which matches with our intuition.

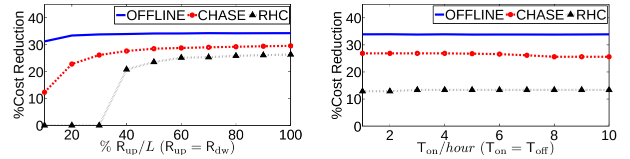
5.5 Impacts of System Parameters

Purpose: Microgrids may employ different types of local generators with diverse operational constraints (such ramping up/down limits and minimum on/off times) and heat recovery efficiencies. It is then important to understand the impact on cost reduction due to these parameters. In this experiment, we study the cost reduction provided by our offline and online algorithms under different settings of R_{up}, R_{dw} , T_{on}, $\mathsf{T}_{\mathrm{off}}$ and η.

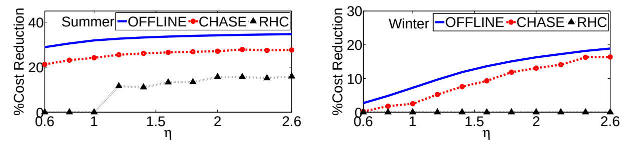
Observations: Fig. 13a and 13b show the impact of ramp limit and minimum on/off time, respectively, on the performance of the algorithms. Note that for simplicity we always set $\mathsf{R}_{\mathrm{up}} = \mathsf{R}_{\mathrm{dw}}$ and $\mathsf{T}_{\mathrm{on}} = \mathsf{T}_{\mathrm{off}}$. As we can see in Fig. 13a, with R_{up} and R_{dw} of about 40% of the maximum capacity, CHASE obtains nearly all of the cost reduction benefits, compared with RHC which needs 70% of the maximum capacity. Meanwhile, it can be seen from Fig. 13b that T_{on} and $\mathsf{T}_{\mathrm{off}}$ do not have much impact on the performance. This suggests that it is more valuable to invest in generators with fast ramping up/down capability than those with small minimum on/off periods. From Fig. 13c and 13d, we observe that generators with large η save much more cost during the winter because of the high heat demand. This suggests that in areas with large heat demand, such as Alaska and Washington, the heat recovery efficiency ratio is a critical parameter when investing CHP generators.

5.6 How Much Local Generation is Enough

Thus far, we assumed that the microgrid had the ability to supply all energy demand from local power generation in every time-slot. In practice, local generators can be quite expensive. Hence, an important question is how much investment should a microgrid operator make (in terms of the installed local generator capacity) in order to obtain the maximum cost benefit. More specifically, we vary the number of CHP generators from 1 to 10 and plot the corresponding cost reductions of algorithms in Fig. 10. Interestingly, our results show that provisioning local generation to produce 60% of the peak demand is sufficient to obtain nearly all of the cost reduction benefits. Further, with just 50% local generation capacity we can achieve about 90% of the



(a) cost redu. v.s. R_{up} and R_{dw} (b) cost redu. v.s. T_{up} and T_{dw}



(c) cost reduction v.s. η (d) cost reduction v.s. η

Figure 13: Cost reduction as functions of generator parameters.

maximum cost reduction. The intuitive reason is that most of the time demands are significantly lower than their peaks.

6. RELATED WORK

Energy generation scheduling is a classical problem in power systems and involves two aspects, namely Unit Commitment (UC) and Economic Dispatching (ED).

For large power systems, UC involves scheduling of a large number gigantic power plants of several hundred if not thousands of megawatts with heterogeneous operating constraints and logistics behind each action [21]. The problem is very challenging to solve and has been shown to be NP-Complete in general [16]. Sophisticated approaches proposed in the literature for solving UC include mix-integer programming [10], dynamic programming [32], and stochastic programming [34]. There have also been investigations on UC with high renewable energy penetration [39, 35], based on over-provisioning approach. After UC determines the on/off status of generators, ED computes their output levels by solving a nonlinear optimization problem using various heuristics without altering the on/off status of generators [11, 31]. There is also recent interest in involving CHP generators in ED to satisfy both electricity and heat demand simultaneously [17]. See comprehensive surveys on UC in [30] and on ED in [14].

However, these studies assume the demand and energy supply (or their distribution) in the entire time horizon are known *a prior*. As such, the schemes are not readily applicable to microgrid scenarios where accurate prediction of small-scale demand and wind power generation is difficult to obtain due to limited management resources and their unpredictable nature [39].

Several recent works have started to study energy generation strategies for microgrids. For example, the authors in [18] develop a linear programming based cost minimization approach for UC in microgrids. [19] considers the fuel consumption rate minimization in microgrids and advocates to build ICT infrastructure in microgrids. The difference between these work and ours is that they assume the demand and energy supply are given beforehand, and ours does not rely on input prediction.

Online optimization and algorithm design is an estab-

lished approach in optimizing the performance of various computer systems with minimum knowledge of inputs [8, 28]. Recently, it has found new applications in data center [15, 9, 36, 24, 25]. To the best of our knowledge, our work is the first to study the competitive online algorithms for energy generation in microgrids with intermittent energy sources and co-generation. The authors in [27] apply online convex optimization framework [40] to design ED algorithms for (micro) grids. However, the ED problem does not take into account the startup cost. In contrast, our work jointly consider UC (with startup cost) and ED in microgrids with co-generation. Further, the two work adopt different frameworks and provide online algorithms with different types of performance guarantee.

7. CONCLUSION

In this paper, we study online algorithms for the microgrid generation scheduling problem with intermittent renewable energy sources and co-generation, with the goal of maximizing the cost-savings with local generation. Based on insights from the structure of the offline optimal solution, we propose a class of competitive online algorithms, called CHASE that track the offline optimal in an online fashion. Under certain settings, we show that CHASE achieves the best competitive ratio of all deterministic online algorithms. We also extend our algorithms to intelligently leverage on *limited prediction* of the future, such as near-term demand or wind forecast. By extensive empirical evaluation using real-world traces, we show that our proposed algorithms can achieve near-offline-optimal performance.

There are a number of interesting directions for future work. First, energy storage systems (*e.g.,* large-capacity battery) have been proposed as an alternate approach to reduce energy generation cost (during peak hours) and to integrate renewable energy sources. It would be interesting to study whether our proposed microgrid control strategies can be combined with energy-storage systems to further reduce generation cost. However, current energy-storage systems can be very expensive. Hence, it is critical to study whether the combined control strategy can reduce sufficient cost with limited amount of energy storage. Second, it remains an open issue whether CHASE can achieve the best competitive ratios in general cases (*e.g.,* in the slow-responding case). There may be potential to obtain further improvement in such cases.

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APPENDIX

A. PROOF OF THEOREM 1

Theorem 1. $(y_{\rm OFA}(t))_{t=1}^T$ is an optimal solution for $\mathbf{SP}$.

PROOF. Suppose $(y^*(t))_{t=1}^T$ is an optimal solution for $\mathbf{SP}$. For completeness, we let $y^*(0)=0$ and $y^*(T+1)=0$. We define a sequence $(y_0(t))_{t=1}^T, (y_1(t))_{t=1}^T, ..., (y_{k+1}(t))_{t=1}^T$ as follows:

1. $y_0(t)=y^*(t)$ for all $t\in[1,T]$.
2. For all $t\in[1,T]$ and $i=1,...,k$

$$y_i(t)=\begin{cases} y_{\rm OFA}(t), & \text{if } t\in[1,T_i^c] \\ y^*(t), & \text{otherwise}\end{cases} \tag{19}$$

3. $y_{k+1}(t)=y_{\rm OFA}(t)$ for all $t\in[1,T]$.

We next set the boundary conditions for each $\mathbf{SP}^{\text{sg-}i}$ by

$$\mathrm{y}_i^l = y_{\rm OFA}(T_i^c) \text{ and } \mathrm{y}_i^r = y^*(T_{i+1}^c+1) \tag{20}$$

It follows that

$$\mathrm{Cost}(y_i)-\mathrm{Cost}(y_{i+1}) = \mathrm{Cost}^{\text{sg-}i}(y^*)-\mathrm{Cost}^{\text{sg-}i}(y_{\rm OFA}) \tag{21}$$

By Lemma 2, we obtain $\mathrm{Cost}^{\text{sg-}i}(y^*)\ge \mathrm{Cost}^{\text{sg-}i}(y_{\rm OFA})$ for all i. Hence,

$$\mathrm{Cost}(y^*)=\mathrm{Cost}(y_0)\ge ...\ge \mathrm{Cost}(y_{k+1})=\mathrm{Cost}(y_{\rm OFA}) \tag{22}$$

□

LEMMA 2. *$(y_{\rm OFA}(t))_{t=T_i^c+1}^{T_{i+1}^c}$ is an optimal solution for $\mathbf{SP}^{\text{sg-}i}(\mathrm{y}_i^l,\mathrm{y}_i^r)$, despite any boundary conditions $(\mathrm{y}_i^l,\mathrm{y}_i^r)$.*

PROOF. Consider given any boundary condition $(\mathrm{y}_i^l,\mathrm{y}_i^r)$ for $\mathbf{SP}^{\text{sg-}i}$. Suppose $(\bar y(t))_{t=T_i^c+1}^{T_{i+1}^c}$ is an optimal solution for $\mathbf{SP}^{\text{sg-}i}$ w.r.t. $(\mathrm{y}_i^l,\mathrm{y}_i^r)$, and $\bar y\ne y_{\rm OFA}$. We aim to show $\mathrm{Cost}^{\text{sg-}i}(\bar y)\ge \mathrm{Cost}^{\text{sg-}i}(y_{\rm OFA})$, by considering the types of critical segment.

(**type-1**): First, suppose that critical segment $[T_i^c+1,T_{i+1}^c]$ is type-1. Hence, $y_{\rm OFA}(t)=1$ for all $t\in[T_i^c+1,T_{i+1}^c]$. Hence,

$$\mathrm{Cost}^{\text{sg-}i}(y_{\rm OFA}) = \beta\cdot(1-\mathrm{y}_i^l)+\sum_{t=T_i^c+1}^{T_{i+1}^c}\psi\big(\sigma(t),1\big) \tag{23}$$

Case 1: Suppose $\bar y(t)=0$ for all $t\in[T_i^c+1,T_{i+1}^c]$. Hence,

$$\mathrm{Cost}^{\text{sg-}i}(\bar y)=\beta\cdot\mathrm{y}_i^r+\sum_{t=T_i^c+1}^{T_{i+1}^c}\psi\big(\sigma(t),0\big) \tag{24}$$

We obtain:

$$\begin{aligned} & \mathrm{Cost}^{\text{sg-}i}(\bar y)-\mathrm{Cost}^{\text{sg-}i}(y_{\rm OFA}) \\ = \;& \beta\cdot\mathrm{y}_i^r+\sum_{t=T_i^c+1}^{T_{i+1}^c}\delta(t)-\beta(1-\mathrm{y}_i^l) \quad (25)\\ \ge \;& \beta\cdot\mathrm{y}_i^r+\Delta(T_{i+1}^c)-\Delta(T_i^c)-\beta(1-\mathrm{y}_i^l) \quad (26)\\ = \;& \beta\cdot\mathrm{y}_i^r+\beta-\beta+\beta\mathrm{y}_i^l\ge 0 \quad (27)\end{aligned}$$

where Eqn. (25) follows from the definition of $\delta(t)$ (see Eqn. (6)) and Eqn. (26) follows from Lemma 3. This completes the proof for Case 1.

(**Case 2**): Suppose $\bar y(t)=1$ for some $t\in[T_i^c+1,T_{i+1}^c]$. This implies that $\mathrm{Cost}^{\text{sg-}i}(\bar y)$ has to involve the startup cost β.

Next, we denote the minimal set of segments within $[T_i^c+1,T_{i+1}^c]$ by

$$[\tau_1^b,\tau_1^e],[\tau_2^b,\tau_2^e],[\tau_3^b,\tau_3^e],...,[\tau_p^b,\tau_p^e]$$

such that $\bar{y}[t] \neq y_{\rm OFA}(t)$ for all $t \in [\tau_j^b, \tau_j^e]$, $j \in \{1, ..., p\}$, where $\tau_j^e < \tau_{j+1}^b$.

Since $\bar{y} \neq y_{\rm OFA}$, then there exists at least one $t \in [T_i^c + 1, T_{i+1}^c]$ such that $\bar{y}(t) = 0$. Hence, τ_1^b is well-defined.

Note that upon exiting each segment $[\tau_j^b, \tau_j^e]$, $\bar{y}$ switches from 0 to 1. Hence, it incurs the startup cost β. However, when $\tau_p^e = T_{i+1}^c$ and $\mathrm{y}_i^r = 0$, the startup cost is not for critical segement $[T_i^c + 1, T_{i+1}^c]$.

Therefore, we obtain:

$$\begin{aligned}
& \mathrm{Cost}^{\text{sg-}i}(\bar{y}) - \mathrm{Cost}^{\text{sg-}i}(y_{\rm OFA}) && (28)\\
= \ & \sum_{t=\tau_1^b}^{\tau_1^e} \delta(t) + \beta - \beta(1 - \mathrm{y}_i^l) + \sum_{j=2}^{p-1} \Big(\sum_{t=\tau_j^b}^{\tau_j^e} \delta(t) + \beta \Big) && (29)\\
& + \sum_{t=\tau_p^b}^{\tau_p^e} \delta(t) + \beta \mathrm{y}_i^r \cdot \mathbf{1}[\tau_p^e = T_{i+1}^c] + \beta \cdot \mathbf{1}[\tau_p^e \neq T_{i+1}^c] \\
\geq \ & \beta - \beta(1 - \mathrm{y}_i^l) \geq 0 && (30)
\end{aligned}$$

where Eqn. (29) follows from the definition of $\delta(t)$.

(**type-2**): Next, suppose that critical segment $[T_i^c+1, T_{i+1}^c]$ is type-2. Hence, $y_{\rm OFA}(t) = 0$ for all $t \in [T_i^c+1, T_{i+1}^c]$. Note that the above argument applies similarly to type-2 setting, when we consider (Case 1): $\bar{y}(t) = 1$ for all $t \in [T_i^c+1, T_{i+1}^c]$ and (Case 2): $\bar{y}(t) = 0$ for some $t \in [T_i^c+1, T_{i+1}^c]$.

(**type-start** and **type-end**): We note that the argument of type-2 applies similarly to type-start and type-end settings.

Therefore, we complete the proof by showing $\mathrm{Cost}^{\text{sg-}i}(\bar{y}) \geq \mathrm{Cost}^{\text{sg-}i}(y_{\rm OFA})$ for all $i \in [0, k]$. $\square$

LEMMA 3. *Suppose $\tau_1, \tau_2 \in [T_i^c + 1, T_{i+1}^c]$ and $\tau_1 < \tau_2$. Then,*

$$\Delta(\tau_2) - \Delta(\tau_1) \begin{cases} \leq \sum_{t=\tau_1+1}^{\tau_2} \delta(t), & \text{if } [T_i^c+1, T_{i+1}^c] \text{ is type-1} \\ \geq \sum_{t=\tau_1+1}^{\tau_2} \delta(t), & \text{if } [T_i^c+1, T_{i+1}^c] \text{ is type-2} \end{cases} \tag{31}$$

PROOF. We recall that

$$\Delta(t) \triangleq \min \Big\{ 0, \max\{-\beta, \Delta(t-1) + \delta(t)\} \Big\} \tag{32}$$

First, we consider $[T_i^c+1, T_{i+1}^c]$ as type-1. This implies that only $\Delta(T_i^c) = -\beta$, whereas $\Delta(t) > -\beta$ for $t \in [T_i^c+1, T_{i+1}^c]$. Hence,

$$\Delta(t) = \min\{0, \Delta(t-1) + \delta(t)\} \leq \Delta(t-1) + \delta(t) \tag{33}$$

Iteratively, we obtain

$$\Delta(\tau_2) \leq \Delta(\tau_1) + \sum_{t=\tau_1+1}^{\tau_2} \delta(t) \tag{34}$$

When $[T_i^c+1, T_{i+1}^c]$ is type-2, we proceed with a similar proof, except

$$\Delta(t) = \max\{-\beta, \Delta(t-1) + \delta(t)\} \geq \Delta(t-1) + \delta(t) \tag{35}$$

Therefore,

$$\Delta(\tau_2) \geq \Delta(\tau_1) + \sum_{t=\tau_1+1}^{\tau_2} \delta(t) \tag{36}$$

$\square$

B. PROOF OF THEOREM 2

Theorem 2. The competitive ratio of $\mathsf{CHASE_s}$

$$\mathsf{CR}(\mathsf{CHASE_s}) \leq 3 - \frac{2(L \cdot c_o + c_m)}{L \cdot (P_{\max} + c_g \cdot \eta)} \tag{37}$$

PROOF. We denote the outcome of $\mathsf{CHASE_s}$ by $(y_{\mathrm{CHASE_s}}(t))_{t=1}^T$. We aim to show that

$$\max_{a,p,h} \frac{\mathrm{Cost}(y_{\mathrm{CHASE_s}})}{\mathrm{Cost}(y_{\rm OFA})} \leq 3 - \frac{2(L \cdot c_o + c_m)}{L \cdot (P_{\max} + c_g \cdot \eta)} \tag{38}$$

First, we denote the set of indexes of critical segments for type-j by $\mathcal{T}_j \subseteq \{0, .., k\}$. Note that we also refer to type-start and type-end by type-0 and type-3 respectively.

Define the sub-cost for type-j by

$$\begin{aligned}
\mathrm{Cost}^{\text{ty-}j}(y) \ \triangleq \ & \sum_{i \in \mathcal{T}_j} \sum_{t=T_i^c+1}^{T_{i+1}^c} \psi\big(\sigma(t), y(t)\big) \\
& + \beta \cdot [y(t) - y(t-1)]^+ && (39)
\end{aligned}$$

Hence, $\mathrm{Cost}(y) = \sum_{j=0}^{3} \mathrm{Cost}^{\text{ty-}j}(y)$. We prove by comparing the sub-cost for each type-j.

(**Type-0**): Note that both $y_{\rm OFA}(t) = y_{\mathrm{CHASE_s}}(t) = 0$ for all $t \in [1, T]$. Hence,

$$\mathrm{Cost}^{\text{ty-}0}(y_{\rm OFA}) = \mathrm{Cost}^{\text{ty-}0}(y_{\mathrm{CHASE_s}}) \tag{40}$$

(**Type-1**): Based on the definition of critical segment (Definition 1), we recall that there is an auxiliary point $\tilde{T}_i^c$, such that either $\big(\Delta(T_i^c) = 0$ and $\Delta(\tilde{T}_i^c) = -\beta\big)$ or $\big(\Delta(T_i^c) = -\beta$ and $\Delta(\tilde{T}_i^c) = 0\big)$.

We also recall that $y_{\rm OFA}(t) = 1$, whereas

$$y_{\mathrm{CHASE_s}}(t) = \begin{cases} 0, & \text{if } t \in [T_i^c+1, \tilde{T}_i^c] \\ 1, & \text{if } t \in [\tilde{T}_i^c, T_{i+1}^c] \end{cases} \tag{41}$$

We consider a particular type-1 critical segment $[T_i^c+1, T_{i+1}^c]$. We note that by the definition of type-1, $y_{\rm OFA}(T_i^c) = y_{\mathrm{CHASE_s}}(T_i^c) = 0$. $y_{\rm OFA}(t)$ and $y_{\mathrm{CHASE_s}}(t)$ switch from 0 to 1 within $[T_i^c + 1, T_{i+1}^c]$, both incurring startup cost β. The cost difference between $y_{\mathrm{CHASE_s}}$ and $y_{\rm OFA}$ within $[T_i^c+1, T_{i+1}^c]$ is

$$\begin{aligned}
& \sum_{t=T_i^c+1}^{\tilde{T}_i^c-1} \Big(\psi\big(\sigma(t), 0\big) - \psi\big(\sigma(t), 1\big) \Big) + \beta - \beta && (42)\\
= \ & \sum_{t=T_i^c+1}^{\tilde{T}_i^c-1} \Big(\psi\big(\sigma(t), 0\big) - \psi\big(\sigma(t), 1\big) \Big) \\
= \ & \sum_{t=T_i^c+1}^{\tilde{T}_i^c-1} \delta(t) = \Delta(\tilde{T}_i^c - 1) - \Delta(T_i^c) && (43)\\
\leq \ & \Delta(\tilde{T}_i^c) - \Delta(T_i^c) = \beta && (44)
\end{aligned}$$

Let the number of type-j critical segments be $m_j \triangleq |\mathcal{T}_j|$.

Then, we obtain

$$\mathrm{Cost}^{\text{ty-}1}(y_{\mathrm{CHASE_s}}) \leq \mathrm{Cost}^{\text{ty-}1}(y_{\rm OFA}) + m_1 \cdot \beta \tag{45}$$

(**Type-2**) and (**Type-3**): We consider a particular type-2 (or type-3) critical segment $[T_i^c+1, T_{i+1}^c]$, we derive similarly for $j = 2$ or 3 as

$$\mathrm{Cost}^{\text{ty-}j}(y_{\mathrm{CHASE_s}}) \leq \mathrm{Cost}^{\text{ty-}j}(y_{\rm OFA}) + m_j \cdot \beta \tag{46}$$

Furthermore, we note $m_1 = m_2 + m_3$, because it takes equal numbers of critical segments for increasing $\Delta(\cdot)$ from $-\beta$ to 0 and for decreasing from 0 to $-\beta$.

Overall, we obtain

$$\begin{aligned}\frac{\text{Cost}(y_{\text{CHASE}_s})}{\text{Cost}(y_{\text{OFA}})} &= \frac{\sum_{j=0}^{3}\text{Cost}^{\text{ty-}j}(y_{\text{CHASE}_s})}{\sum_{j=0}^{3}\text{Cost}^{\text{ty-}j}(y_{\text{OFA}})} \\ &\le \frac{(m_1+m_2+m_3)\beta + \sum_{j=0}^{3}\text{Cost}^{\text{ty-}j}(y_{\text{OFA}})}{\sum_{j=0}^{3}\text{Cost}^{\text{ty-}j}(y_{\text{OFA}})} \\ &\le 1 + \frac{2m_1\beta}{\sum_{j=0}^{3}\text{Cost}^{\text{ty-}j}(y_{\text{OFA}})} \\ &\le 1 + \begin{cases} 0 & \text{if } m_1 = 0, \\ \dfrac{2m_1\beta}{\text{Cost}^{\text{ty-}1}(y_{\text{OFA}})} & \text{otherwise} \end{cases}\end{aligned} \tag{47}$$

By Lemma 4 and simplications, we obtain

$$\frac{\text{Cost}(y_{\text{CHASE}_s})}{\text{Cost}(y_{\text{OFA}})} \le 3 - \frac{2(L\cdot c_o + c_m)}{L\cdot\big(P_{\max} + \eta\cdot c_g\big)} \tag{48}$$

□

LEMMA 4.

$$\text{Cost}^{\text{ty-}1}(y_{\text{OFA}}) \ge m_1\Big(\beta + \beta\frac{L\cdot c_o + c_m}{L\big(P_{\max}+\eta\cdot c_g - c_o\big) - c_m}\Big) \tag{49}$$

PROOF. Considering Type-1 critical segment, we have

$$\text{Cost}^{\text{ty-}1}(y_{\text{OFA}}) = \sum_{i\in\mathcal{T}_1}\Big(\beta + \sum_{t=T_i^c+1}^{T_{i+1}^c}\psi\big(\sigma(t),1\big)\Big) \tag{50}$$

On the other hand, we obtain

$$\begin{aligned}&\sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),1\big) - c_m\Big) \\ &= \frac{\sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),1\big) - c_m\Big)}{\sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\Big)} \\ &\quad\times\sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\Big) \\ &\ge \min_{\tau\in[T_i^c+1,T_{i+1}^c]}\frac{\psi\big(\sigma(\tau),1\big) - c_m}{\psi\big(\sigma(\tau),0\big) - \psi\big(\sigma(\tau),1\big) + c_m} \\ &\quad\times\sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\Big)\end{aligned} \tag{51}$$

whereas Lemma 5 shows

$$\min_{\tau\in[T_i^c+1,T_{i+1}^c]}\frac{\psi\big(\sigma(\tau),1\big) - c_m}{\psi\big(\sigma(\tau),0\big) - \psi\big(\sigma(\tau),1\big) + c_m} \ge \frac{c_o}{P_{\max}+\eta\cdot c_g - c_o} \tag{52}$$

and

$$\begin{aligned}&\sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\Big) \\ &= \sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big)\Big) + (T_{i+1}^c - T_i^c)c_m \\ &\ge \beta + (T_{i+1}^c - T_i^c)c_m\end{aligned} \tag{53}$$

Furthermore, we note that $T_{i+1}^c - T_i^c$ is lower bounded by the steepest descend when $p(t) = P_{\max}$, $a(t) = L$ and $h(t) = \eta L$ for all $t\in[T_i^c+1, T_{i+1}^c]$:

$$T_{i+1}^c - T_i^c \ge \frac{\beta}{L\cdot\big(P_{\max}+\eta\cdot c_g - c_o\big) - c_m} \tag{54}$$

Together, we obtain:

$$\begin{aligned}&\sum_{t=T_i^c+1}^{T_{i+1}^c}\psi\big(\sigma(t),1\big) \\ &= \sum_{t=T_i^c+1}^{T_{i+1}^c}\Big(\psi\big(\sigma(t),1\big) - c_m\Big) + \sum_{t=T_i^c+1}^{T_{i+1}^c}c_m \\ &\ge \frac{c_o\big(\beta + (T_{i+1}^c - T_i^c)c_m\big)}{P_{\max}+\eta\cdot c_g - c_o} + (T_{i+1}^c - T_i^c)c_m \\ &\ge \beta\frac{L\cdot c_o + c_m}{L\cdot\big(P_{\max}+\eta\cdot c_g - c_o\big) - c_m}\end{aligned}$$

Therefore,

$$\begin{aligned}&\text{Cost}^{\text{ty-}1}(y_{\text{OFA}}) \\ &= \textstyle\sum_{i\in\mathcal{T}_1}\Big(\beta + \sum_{t=T_i^c+1}^{T_{i+1}^c}\big(\psi\big(\sigma(t),1\big)\big)\Big) \\ &\ge m_1\Big(\beta + \beta\frac{L\cdot c_o + c_m}{L\cdot\big(P_{\max}+\eta\cdot c_g - c_o\big) - c_m}\Big)\end{aligned} \tag{55}$$

□

LEMMA 5.

$$\min_{\tau\in[T_i^c+1,T_{i+1}^c]}\frac{\psi\big(\sigma(\tau),1\big) - c_m}{\psi\big(\sigma(\tau),0\big) - \psi\big(\sigma(\tau),1\big) + c_m} \ge \frac{c_o}{P_{\max}+\eta\cdot c_g - c_o} \tag{56}$$

PROOF. We expand $\psi(\sigma(t), y(t))$ for each case:

Case 1: $c_o \ge p(t) + \eta\cdot c_g$. When $y(t) = 1$, by Lemma 1

$$u(t) = 0, v(t) = a(t), s(t) = h(t) \tag{57}$$

Thus,

$$\psi\big(\sigma(\tau),1\big) = p(t)a(t) + c_g s(t) + c_m \tag{58}$$
$$\psi\big(\sigma(\tau),0\big) = p(t)a(t) + c_g s(t) \tag{59}$$

Therefore,

$$\frac{\psi\big(\sigma(\tau),1\big) - c_m}{\psi\big(\sigma(\tau),0\big) - \psi\big(\sigma(\tau),1\big) + c_m} = \infty \tag{60}$$

Case 2: $c_o \le p(t)$. When $y(t) = 1$, by Lemma 1

$$u(t) = a(t), v(t) = 0, s(t) = \big[h(t) - \eta\cdot a(t)\big]^+ \tag{61}$$

Thus,

$$\psi\big(\sigma(\tau),1\big) = c_o a(t) + c_g\big[h(t) - \eta\cdot a(t)\big]^+ + c_m \tag{62}$$
$$\psi\big(\sigma(\tau),0\big) = p(t)a(t) + c_g h(t) \tag{63}$$

Therefore,

$$\begin{aligned}
& \frac{\psi(\sigma(\tau),1)-c_m}{\psi(\sigma(\tau),0)-\psi(\sigma(\tau),1)+c_m} \\
= & \frac{c_o a(t)+c_g\left[h(t)-\eta\cdot a(t)\right]^+}{\left(p(t)-c_o\right)a(t)+c_g\left(h(t)-\left[h(t)-\eta\cdot a(t)\right]^+\right)} \\
\geq & \frac{c_o a(t)}{\left(p(t)-c_o\right)a(t)+c_g\min\{h(t),\eta\cdot a(t)\}} \\
\geq & \frac{c_o a(t)}{\left(p(t)-c_o\right)a(t)+\eta\cdot c_g a(t)} \\
= & \frac{c_o}{p(t)-c_o+\eta\cdot c_g}
\end{aligned}$$

Case 3: $p(t)+\eta\cdot c_g > c_o > p(t)$. When $y(t)=1$, by Lemma 1

$$u(t) = \min\left\{\frac{h(t)}{\eta}, a(t)\right\}, \tag{64}$$

$$v(t) = \max\left\{0, a(t)-\frac{h(t)}{\eta}\right\}, \tag{65}$$

$$s(t) = \max\left\{0, h(t)-\eta\cdot a(t)\right\} \tag{66}$$

Thus,

$$\begin{aligned}
\psi(\sigma(\tau),1) &= c_o\min\left\{\frac{h(t)}{\eta},a(t)\right\}+p(t)\left[a(t)-\frac{h(t)}{\eta}\right]^+ \\
&\quad +c_g\left[h(t)-\eta\cdot a(t)\right]^+ + c_m \qquad (67)\\
\psi(\sigma(\tau),0) &= p(t)a(t)+c_g h(t) \qquad (68)
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\psi(\sigma(\tau),0)-\psi(\sigma(\tau),1)+c_m \\
&= p(t)\min\left\{a(t),\tfrac{h(t)}{\eta}\right\}+c_g\min\left\{h(t),\eta\cdot a(t)\right\}-c_o\min\left\{\tfrac{h(t)}{\eta},a(t)\right\}
\end{aligned}$$

and

$$\begin{aligned}
& \frac{\psi(\sigma(\tau),1)-c_m}{\psi(\sigma(\tau),0)-\psi(\sigma(\tau),1)+c_m} \\
= & \begin{cases} \dfrac{c_o a(t)+c_g\left(h(t)-\eta\cdot a(t)\right)}{p(t)a(t)+c_g\eta\cdot a(t)-c_o a(t)} & \text{if } h(t)-\eta\cdot a(t)>0 \\ \dfrac{c_o\frac{h(t)}{\eta}+p(t)\left(a(t)-\frac{h(t)}{\eta}\right)}{p(t)\frac{h(t)}{\eta}+c_g h(t)-c_o\frac{h(t)}{\eta}} & \text{otherwise} \end{cases} \\
\geq & \frac{c_o}{p(t)+\eta\cdot c_g-c_o} \qquad (69)
\end{aligned}$$

Combing all the cases, we obtain

$$\min_{\tau\in[T_i^c+1,T_{i+1}^c]}\frac{\psi(\sigma(\tau),1)-c_m}{\psi(\sigma(\tau),0)-\psi(\sigma(\tau),1)+c_m}\geq\frac{c_o}{P_{\max}+\eta\cdot c_g-c_o}$$

$\square$

C. PROOF OF THEOREM 3

LEMMA 6. *Denote an online algorithm by $\mathcal{A}$, and an input sequence by $\sigma=(a(t),h(t),p(t))_{t=1}^T$. More specifically, we write $\mathrm{Cost}(y_{\mathcal{A}},u_{\mathcal{A}},v_{\mathcal{A}};\sigma)$ and $\mathrm{Cost}(y_{\mathcal{A}};\sigma)$, when it explicitly refers to input sequence by σ. Define*

$$\underline{c}(\mathbf{fMCMP})\triangleq\min_{\mathcal{A}}\max_{\sigma}\frac{\mathrm{Cost}(y_{\mathcal{A}},u_{\mathcal{A}},v_{\mathcal{A}},s_{\mathcal{A}};\sigma)}{\mathrm{Cost}(y_{\mathrm{OFA}},u_{\mathrm{OFA}},v_{\mathrm{OFA}},s_{\mathrm{OFA}};\sigma)} \tag{70}$$

$$\underline{c}(\mathbf{SP})\triangleq\min_{\mathcal{A}}\max_{\sigma}\frac{\mathrm{Cost}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}(y_{\mathrm{OFA}};\sigma)} \tag{71}$$

We have

$$\underline{c}(\mathbf{SP})\leq\underline{c}(\mathbf{fMCMP}) \tag{72}$$

PROOF. We prove this lemma by contradiction. Suppose that there exists a deterministic online algorithm $\mathcal{A}$ for **fMCMP** with output $(y_{\mathcal{A}},u_{\mathcal{A}},v_{\mathcal{A}},s_{\mathcal{A}})$, such that

$$\max_{\sigma}\frac{\mathrm{Cost}(y_{\mathcal{A}},u_{\mathcal{A}},v_{\mathcal{A}},s_{\mathcal{A}};\sigma)}{\mathrm{Cost}(y_{\mathrm{OFA}},u_{\mathrm{OFA}},v_{\mathrm{OFA}},s_{\mathrm{OFA}};\sigma)}<\underline{c}(\mathbf{SP}) \tag{73}$$

Also, it follows that for any an input sequence σ',

$$\begin{aligned}
&\frac{\mathrm{Cost}(y_{\mathcal{A}},u_{\mathcal{A}},v_{\mathcal{A}},s_{\mathcal{A}};\sigma')}{\mathrm{Cost}(y_{\mathrm{OFA}},u_{\mathrm{OFA}},v_{\mathrm{OFA}},s_{\mathrm{OFA}};\sigma')} \qquad (74)\\
\leq\ &\max_{\sigma}\frac{\mathrm{Cost}(y_{\mathcal{A}},u_{\mathcal{A}},v_{\mathcal{A}},s_{\mathcal{A}};\sigma)}{\mathrm{Cost}(y_{\mathrm{OFA}},u_{\mathrm{OFA}},v_{\mathrm{OFA}},s_{\mathrm{OFA}};\sigma)} \qquad (75)
\end{aligned}$$

It follows that (by Lemma 1)

$$\mathrm{Cost}(y_{\mathcal{A}};\sigma)\leq\mathrm{Cost}(y_{\mathcal{A}},u_{\mathcal{A}},v_{\mathcal{A}};\sigma) \tag{76}$$

Based on $\mathcal{A}$, we can construct an online algorithm $\mathcal{A}'$ for **SP**, such that $y_{\mathcal{A}'}=y_{\mathcal{A}}$. By Lemma 1,

$$\mathrm{Cost}(y_{\mathrm{OFA}};\sigma)=\mathrm{Cost}(y_{\mathrm{OFA}},u_{\mathrm{OFA}},v_{\mathrm{OFA}};\sigma) \tag{77}$$

Therefore, we obtain

$$\frac{\mathrm{Cost}(y_{\mathcal{A}};\sigma')}{\mathrm{Cost}(y_{\mathrm{OFA}};\sigma')}<\underline{c}(\mathbf{SP}) \tag{78}$$

However, as $\underline{c}(\mathbf{SP})$ is a lower bound of competitive ratio for any deterministic online algorithm for **SP**. This is contradiction, and it completes our proof. $\square$

Theorem 3. The competitive ratio for any deterministic online algorithm $\mathcal{A}$ for **SP** is lower bounded by a function:

$$\mathsf{CR}(\mathcal{A})\geq\mathsf{cr}(\beta) \tag{79}$$

When $\frac{\beta}{L(P_{\max}+\eta\cdot c_g-c_o)}\to\infty$, we have

$$\mathsf{cr}(\beta)\to\min\left\{\frac{L\cdot P_{\max}}{L\cdot c_o+c_m},3-\frac{2(L\cdot c_o+c_m)}{L\cdot\left(P_{\max}+c_g\cdot\eta\right)}\right\} \tag{80}$$

PROOF. The basic idea is as follows. Given any deterministic online algorithm $\mathcal{A}$, we construct a special input sequence σ, such that

$$\frac{\mathrm{Cost}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}(y_{\mathrm{OFA}};\sigma)}\geq\mathsf{cr}(\beta) \tag{81}$$

for a function $\mathsf{cr}(\beta)$.

First, we note that at time t, $\mathcal{A}$ determines $y_{\mathcal{A}}(t)$ only based on the past input in $[1,t-1]$. Thus, we construct an input sequence $\sigma=(a(t),h(t),p(t))_{t=1}^T$ progressively as follows:

- $p(t)=P_{\max}$.
- $a(t)=L\cdot(1-y_{\mathcal{A}}(t-1))$. Namely,

$$a(t)=\begin{cases}L, & \text{if } y_{\mathcal{A}}(t-1)=0\\ 0, & \text{if } y_{\mathcal{A}}(t-1)=1\end{cases} \tag{82}$$

- $h(t)=\eta\cdot a(t)$.

For completeness, we set the boundary conditions: $p(0) = p(T+1) = 0$ and $a(0) = a(T+1) = 0$.

Step 1: Computing $\mathrm{Cost}(y_{\mathcal{A}};\sigma)$:

Based on our construction of $(\sigma(t))_{t=1}^{T}$, we can partition $[1,T]$ into disjoint segments of consecutive intervals of full demand or zero demand:

- Full-demand segment: $[t_1,t_2]$, if $a(t) = L$ for all $t \in [t_1,t_2]$, and $a(t_1-1) = a(t_2+1) = 0$.
- Zero-demand segment: $[t_1,t_2]$, if $a(t) = 0$ for all $t \in [t_1,t_2]$, and $a(t_1-1) = a(t_2+1) = L$.

Note that according to Eqn. (82), $a(1) = L$. Thus, the time $t = 1$ must belong to a full-demand segment. Also, full-demand and zero-demand segments appear alternating.

Let n_f and n_z be the number of full-demand and zero-demand segments in $[1,T]$ respectively. Thus, we have

$$n_z \le n_f \le n_z + 1 \tag{83}$$

In a full-demand segment $[t_1,t_2]$, since $a(t) = L$, for all $t \in [t_1,t_2]$ and according to the construction of $a(t)$ in Eqn. (82), we conclude that $y(t)$ generated by $\mathcal{A}$ must be

$$y_{\mathcal{A}}(t) = \begin{cases} 0, & t_1 \le t \le t_2 - 1 \\ 1, & t = t_2 \end{cases} \tag{84}$$

As a result, in a full-demand segment $[t_1,t_2]$ with length (t_2-t_1+1), $\mathcal{A}$ incurs a cost

$$L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\big(t_2-t_1\big)+\beta+(L\cdot c_o+c_m)\cdot 1. \tag{85}$$

Similarly, in a zero-demand segment $[t_1,t_2]$, $\mathcal{A}$ incurs a cost

$$\big(t_2-t_1\big)c_m. \tag{86}$$

Let Σ_f and Σ_z be the total lengths of full-demand and zero-demand segments in $[1,T]$ respectively. By summing the costs over all full-demand and zero-demand segments and simplifying terms, we obtain a compact expression of the cost of $\mathcal{A}$ w.r.t. σ as follows:

$$\begin{aligned} & \mathrm{Cost}(y_{\mathcal{A}};\sigma) \\ = \; & L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot(\Sigma_f-n_f)+n_f\big(\beta+L\cdot c_o+c_m\big) \\ & +c_m\cdot(\Sigma_z-n_z) \\ = \; & L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\Sigma_f+n_f\Big(\beta+L\cdot c_o+c_m \\ & -L\cdot\big(P_{\max}+\eta\cdot c_g\big)\Big)+c_m\Sigma_z-c_m n_z \end{aligned} \tag{87}$$

Step 2: (Bounding $\mathrm{Cost}(y_{\mathcal{A}};\sigma)/\mathrm{Cost}(y_{\mathrm{OFA}};\sigma)$):

We divide the input σ into critical segments. We then define $\mathcal{S}_{up}$ be the set of all type-0, type-2, type-3, and the "increasing" parts of type-1 critical segments, and $\mathcal{S}_{pt}$ be set of the "plateau" parts of type-1 critical segments.

Here, for a type-1 critical segment $[T_i^c+1, T_{i+1}^c]$, the "increasing" part is defined as $[T_i^c+1, \tilde{T}_i^c]$ and the "plateau" part is defined as $[\tilde{T}_i^c+1, T_{i+1}^c]$. We define $\mathrm{Cost}_{up}(\cdot)$ and $\mathrm{Cost}_{pt}(\cdot)$ be the costs for $\mathcal{S}_{up}$ and $\mathcal{S}_{pt}$ respectively.

On the increasing part $[T_i^c+1, \tilde{T}_i^c]$, the deficit function wriggles up from $-\beta$ to 0, and it cost the same to served the part by either buying power from the grid or using on-site generator (which incurs a turning-on cost). Hence, we can simplify the offline cost on the increasing part as

$$\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma) = \beta + \sum_{t=T_i^c+1}^{\tilde{T}_i^c}\psi\big(\sigma(t),1\big) = \sum_{t=T_i^c+1}^{\tilde{T}_i^c}\psi\big(\sigma(t),0\big).$$

With this simplification, we proceed with the ratio analysis as follows:

$$\begin{aligned} \frac{\mathrm{Cost}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}(y_{\mathrm{OFA}};\sigma)} & = \frac{\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)+\mathrm{Cost}_{pt}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)+\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma)} \\ & \ge \min\Big\{\frac{\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)}, \; \frac{\mathrm{Cost}_{pt}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma)}\Big\} \end{aligned}$$

As T goes to infinity, it is clear that to lower-bound the above ratio, it suffices to consider only those $\mathcal{S}_i$ ($i \in \{up, pt\}$) with unbounded length in time. Next, we study each term in the lower bound of the competitive ratio. We define Σ_f^{up} (Σ_z^{up}) and Σ_f^{pt} (Σ_z^{pt}) as the total length of full-demand (zero-demand) intervals in the increasing parts and plateau, respectively. Similarly, we define n_f^{up} (n_z^{up}) and n_f^{pt} (n_z^{pt}) as the number of full-demand (zero-demand) intervals in the increasing parts and plateau, respectively.

Step 2-1: (Bounding $\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)/\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)$)

First, we seek to lower-bound the term $\frac{\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)}$ under the assumption that $|\mathcal{S}_{up}|$ is unbounded. From the offline solution structure, we know that on type-0, type-2, type-3, and the "increasing" parts of type-1 critical segments, the offline optimal cost is given by

$$\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma) = L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\Sigma_f^{up} \tag{88}$$

Noticing that we also have $n_f^{up} \ge n_z^{up}$, we obtain

$$\begin{aligned} & \frac{\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)} \\ = \; & \Big(L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\Sigma_f^{up}+\big(\beta+L\cdot c_o \\ & -L\big(P_{\max}+\eta\cdot c_g\big)+c_m\big)n_f^{up}+c_m\cdot\Sigma_z^{up}-c_m n_z^{up}\Big)\Big/ \\ & \Big(L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\Sigma_f^{up}\Big) \\ \ge \; & \Big(L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\Sigma_f^{up}+\big(\beta+L\cdot c_o \\ & -L\big(P_{\max}+\eta\cdot c_g\big)\big)n_f^{up}+c_m\cdot\Sigma_z^{up}\Big)\Big/ \\ & \Big(L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\Sigma_f^{up}\Big) \end{aligned}$$

In $\mathcal{S}_{up}$, either there is only one type-0 segment, or there are equal number of type-2/3 critical segments and type-1 critical segment "increasing" parts. Hence, the total deficit function increment, i.e., $(LP_{\max}-L\cdot c_o-c_m)\Sigma_f^{up}$, must be no more than the total deficit function decrement, which is upper bounded by $c_m\Sigma_z^{up}+\beta$ where the term β accounts for that the deficit function does not end naturally at but get dragged down to$-\beta$ at the end of T. That is,

$$c_m\Sigma_z^{up}+\beta-\big(L\big(P_{\max}+\eta\cdot c_g-c_o\big)-c_m\big)\Sigma_f^{up} \ge 0. \tag{89}$$

Moreover, since $\mathcal{S}_{up}$ contains only type-0, type-2, type-3, and the "increasing" parts of type-1 critical segments, the deficit function increment introduced by every full-demand segment must be no more than β. That is,

$$n_f^{up}\beta \ge \big(L\big(P_{\max}+\eta\cdot c_g-c_o\big)-c_m\big)\Sigma_f^{up}. \tag{90}$$

By the above inequalities, we continue the derivation as follows:

$$\begin{aligned} \frac{\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)} \ge \; & 1+\frac{\beta+L\cdot(c_o-P_{\max}-\eta\cdot c_g)}{L\cdot\big(P_{\max}+\eta\cdot c_g\big)\Sigma_f^{up}/n_f^{up}} \\ & +\frac{c_m\Sigma_z^{up}}{L\cdot\big(P_{\max}+\eta\cdot c_g\big)\cdot\Sigma_f^{up}} \end{aligned} \tag{91}$$

Now we discuss the second term in Eqn. (91), denoted as (I). Recall in the problem setting, $\beta + L\cdot c_o - L\cdot\big(P_{\max} + \eta\cdot c_g\big) > 0$. Hence, term (I) is monotonically decreasing in Σ_f^{up}/n_f^{up}, and its minimum value is taken when Σ_f^{up}/n_f^{up} is replaced with the upper-boundary value $\beta(LP_{\max} - L\cdot c_o - c_m)^{-1}$:

$$\begin{aligned}
& (I)\\
\geq\ & (\beta + L\cdot(c_o - P_{\max} - \eta\cdot c_g)) \,/\, \big(L\cdot\big(P_{\max} + \eta\cdot c_g\big)\\
& \cdot\beta\cdot\big(L\cdot\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m\big)^{-1}\big)\\
=\ & \frac{L\cdot\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m}{L\cdot\big(P_{\max} + \eta\cdot c_g\big)} \qquad (92)
\end{aligned}$$

The above inequality holds for arbitrary $|\mathcal{S}_{up}|$.

Now we discuss the third term in Eqn. (91), denoted as (II). When $|\mathcal{S}_{up}|$ goes to infinity, (II) can be discussed by two cases. In the first case, Σ_f^{up} remains bounded when $|\mathcal{S}_{up}|$ goes to infinity. Since $\Sigma_f^{up} + \Sigma_z^{up} = |\mathcal{S}_{up}|$ we must have unbounded Σ_z^{up} as $|\mathcal{S}_{up}|$ goes to infinity. As a result, the term (II) is unbounded, and so is $\frac{\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)}$. In the second case, Σ_f^{up} goes unbounded when $|\mathcal{S}_{up}|$ goes to infinity. Then by Eqn. (89), we know that Σ_z^{up} also goes unbounded and,

$$\begin{aligned}
(II)\ \geq\ & \frac{c_m\cdot\Sigma_z^{up}}{L\cdot\big(P_{\max} + \eta\cdot c_g\big)}\cdot\frac{L\cdot\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m}{c_m\cdot\Sigma_z^{up} + \beta}\\
=\ & \frac{L\cdot\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m}{L\cdot\big(P_{\max} + \eta\cdot c_g\big)}\cdot\frac{c_m\cdot\Sigma_z^{up}}{c_m\cdot\Sigma_z^{up} + \beta}\\
=\ & \frac{L\cdot\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m}{L\cdot\big(P_{\max} + \eta\cdot c_g\big)} (\text{as } |\mathcal{S}_{up}| \to \infty) \quad (93)
\end{aligned}$$

Overall, by substituting Eqn. (92) and Eqn. (93) into Eqn. (91), we obtain

$$\frac{\mathrm{Cost}_{up}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{up}(y_{\mathrm{OFA}};\sigma)} \ \geq\ 3 - 2\frac{L\cdot c_o + c_m}{L\cdot\big(P_{\max} + \eta\cdot c_g\big)}. \quad (94)$$

Step 2-2: (Bounding $\mathrm{Cost}_{pt}(y_{\mathcal{A}};\sigma)/\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma)$)

We now lower-bound the term $\frac{\mathrm{Cost}_{pt}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma)}$ under the assumption that $|\mathcal{S}_{pt}|$ is unbounded. Since $\mathcal{S}_{pt}$ only contains the "plateau" parts of type-1 critical segments, we have

$$\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma) = (L\cdot c_o + c_m)\Sigma_f^{pt} + c_m\Sigma_z^{pt} \quad (95)$$

Therefore,

$$\begin{aligned}
& \frac{\mathrm{Cost}_{pt}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma)}\\
=\ & \Big(L\cdot\big(P_{\max} + \eta\cdot c_g\big)\cdot\Sigma_f^{pt} + \big(\beta + L\cdot c_o\\
& - L\big(P_{\max} + \eta\cdot c_g\big) + c_m\big)n_f^{pt} + c_m\cdot\Sigma_z^{pt} - c_m n_z^{pt}\Big)\Big/\\
& \Big((L\cdot c_o + c_m)\Sigma_f^{pt} + c_m\Sigma_z^{pt}\Big)
\end{aligned}$$

On $\mathcal{S}_{pt}$, the total deficit function increment, i.e., $(L\big(P_{\max} + \eta\cdot c_g\big) - L\cdot c_o - c_m)\Sigma_f^{pt}$, must be no less than the total deficit function decrement, which is $c_m\Sigma_z^{pt}$. That is,

$$\big(L\cdot\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m\big)\Sigma_f^{pt} \geq c_m\Sigma_z^{pt}. \quad (96)$$

Moreover, on $\mathcal{S}_{pt}$, the deficit function decrement caused by every zero-demand segment must be less than β; otherwise, the deficit function will reach value $-\beta$ and it cannot be a "plateau" part of a type-1 critical segment. That is,

$$n_z^{pt}\beta \geq c_m\Sigma_z^{pt} \quad (97)$$

Since a plateau part of a type-1 critical segment must end with a full-demand interval, we must have $n_f^{pt} \geq n_z^{pt}$.

We continue the lower bound analysis as follows:

$$\begin{aligned}
& \frac{\mathrm{Cost}_{pt}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma)}\\
\geq\ & \frac{L\cdot\big(P_{\max} + \eta\cdot c_g\big)\cdot\Sigma_f^{pt} + c_m\cdot\Sigma_z^{pt}}{(L\cdot c_o + c_m)\Sigma_f^{pt} + c_m\Sigma_z^{pt}}\\
& + \frac{\big(\beta + L\cdot c_o - L\big(P_{\max} + \eta\cdot c_g\big)\big)n_f^{pt}}{(L\cdot c_o + c_m)\Sigma_f^{pt} + c_m\Sigma_z^{pt}}\\
\geq\ & \frac{L\cdot P_{\max} + \big(1 + \frac{\beta + L\cdot(c_o - P_{\max} - \eta\cdot c_g)}{\beta}\big)c_m\cdot\Sigma_z^{pt}/\Sigma_f^{pt}}{L\cdot c_o + c_m + c_m\Sigma_z^{pt}/\Sigma_f^{pt}}.
\end{aligned}$$

By checking the derivative, we know that the last term is monotonically increasing/decreasing in the ratio $c_m\cdot\Sigma_z^{pt}/\Sigma_f^{pt}$. Hence, its minimum value is taken when the ratio is replaced with the lower-boundary value 0 or the upper-boundary value $L\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m$. Carrying out the derivation and taking into account the problem setting $\frac{\beta}{L(P_{\max} + \eta\cdot c_g - c_o)} \to \infty$, we obtain

$$\begin{aligned}
& \frac{\mathrm{Cost}_{pt}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}_{pt}(y_{\mathrm{OFA}};\sigma)}\\
\geq\ & \min\Big\{\frac{L\big(P_{\max} + \eta\cdot c_g\big)}{L\cdot c_o + c_m}, 3 - 2\frac{L\cdot c_o + c_m}{L\big(P_{\max} + \eta\cdot c_g\big)}\Big\}.
\end{aligned}$$

At the end, we obtain the desired result of

$$\frac{\mathrm{Cost}(y_{\mathcal{A}};\sigma)}{\mathrm{Cost}(y_{\mathrm{OFA}};\sigma)} \geq \min\Big\{\ \frac{L\big(P_{\max} + \eta\cdot c_g\big)}{L\cdot c_o + c_m},\quad 3 - 2\frac{L\cdot c_o + c_m}{L\big(P_{\max} + \eta\cdot c_g\big)}\ \Big\}. \quad (98)$$

□

D. PROOF OF THEOREM 4

Theorem 4. The competitive ratio of $\mathsf{CHASE}_s^{lk(\omega)}$

$$\begin{aligned}
& \mathsf{CR}(\mathsf{CHASE}_s^{\mathrm{lk}(w)}) \quad (99)\\
\leq\ & 1 + \frac{2\beta L\big(P_{\max} + \eta\cdot c_g - c_o - \frac{c_m}{L}\big)}{\big(P_{\max} + \eta\cdot c_g\big)\big(\beta L + w\cdot c_m\big(L - \frac{c_m}{P_{\max} + \eta\cdot c_g - c_o}\big)\big)}
\end{aligned}$$

Proof. We note that the proof is similar that of Theorem 2, except with modifications considering time window w. We denote the outcome of $\mathsf{CHASE}_s^{\mathrm{lk}(w)}$ by $\big(y_{\mathsf{CHASE}(w)}(t)\big)_{t=1}^T$.

(**type-0**): Similar to the proof of Theorem 2

(**type-1**): Based on the definition of critical segment (Definition 1), we recall that there is an auxiliary point $\tilde{T}_i^c$, such that either $\big(\Delta(T_i^c) = 0$ and $\Delta(\tilde{T}_i^c) = -\beta\big)$ or $\big(\Delta(T_i^c) = -\beta$ and $\Delta(\tilde{T}_i^c) = 0\big)$. We focus on the segement $T_i^c + 1 + w < \tilde{T}_i^c$. We observe

$$y_{\mathsf{CHASE}(w)}(t) = \begin{cases} 0, & \text{for all } t \in [T_i^c + 1, \tilde{T}_i^c - w),\\ 1, & \text{for all } t \in [\tilde{T}_i^c - w, T_{i+1}^c], \end{cases} \quad (100)$$

We consider a particular type-1 critical segment, i.e., kth type-1 critical segment: $[T_i^c+1, T_{i+1}^c]$. Note that by the definition of type-1, $y_{\text{OFA}}(T_i^c) = y_{\mathsf{CHASE}(w)}(T_i^c) = 0$. $y_{\text{OFA}}(t)$ switches from 0 to 1 at time T_i^c+1, while $y_{\mathsf{CHASE}(w)}$ switches at time $\tilde{T}_i^c - w - 1$, both incurring startup cost β. The cost difference between y_{CHASE} and y_{OFA} within $[T_i^c+1, T_{i+1}^c]$ is

$$\sum_{t=T_i^c+1}^{\tilde{T}_i^c-1} \Big(\psi\Big(\sigma(t),0\Big) - \psi\Big(\sigma(t),1\Big)\Big) + \beta - \beta \tag{101}$$

$$= \sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1} \delta(t) = \Delta(\tilde{T}_i^c - w - 1) - \Delta(T_i^c) = q_i^1 + \beta$$

where $q_i^1 \triangleq \Delta(\tilde{T}_i^c - w - 1)$.

Recall the number of type-j critical segments $m_j \triangleq |\mathcal{T}_j|$.

$$\text{Cost}^{\text{ty-1}}(y_{\mathsf{CHASE}(w)}) \le \text{Cost}^{\text{ty-1}}(y_{\text{OFA}}) + m_1 \cdot \beta + \sum_{i=1}^{m_1} q_i^1 \tag{102}$$

(**type-2**) and (**type-3**): We derive similarly for $j = 2$ or 3 as

$$\text{Cost}^{\text{ty-}j}(y_{\mathsf{CHASE}(w)}) \le \text{Cost}^{\text{ty-}j}(y_{\text{OFA}}) - \sum_{i=1}^{m_j} q_i^j. \tag{103}$$

Note that $|q_i^j| \le \beta$ for all i, j.

Furthermore, we note $m_1 = m_2 + m_3$. Overall, we obtain

$$\frac{\text{Cost}(y_{\mathsf{CHASE}(w)})}{\text{Cost}(y_{\text{OFA}})} = \frac{\sum_{j=0}^{3} \text{Cost}^{\text{ty-}j}(y_{\mathsf{CHASE}(w)})}{\sum_{j=0}^{3} \text{Cost}^{\text{ty-}j}(y_{\text{OFA}})}$$
$$\le \frac{m_1\beta + \sum_{k=1}^{m_1} q_i^1 + (m_2+m_3)\beta + \sum_{j=0}^{3} \text{Cost}^{\text{ty-}j}(y_{\text{OFA}})}{\sum_{j=0}^{3} \text{Cost}^{\text{ty-}j}(y_{\text{OFA}})}$$
$$= 1 + \frac{2m_1\beta + \sum_{k=1}^{m_1} q_i^1}{\sum_{j=0}^{3} \text{Cost}^{\text{ty-}j}(y_{\text{OFA}})}$$
$$\le 1 + \begin{cases} 0 & \text{if } m_1 = 0, \\ \dfrac{2m_1\beta + \sum_{k=1}^{m_1} q_i^1}{\text{Cost}^{\text{ty-1}}(y_{\text{OFA}})} & \text{otherwise} \end{cases}$$

By Lemma 7 and simplifications, we obtain

$$\frac{\text{Cost}(y_{\mathsf{CHASE}(w)})}{\text{Cost}(y_{\text{OFA}})} \tag{104}$$
$$\le 1 + \frac{2\beta L\big(P_{\max} + \eta\cdot c_g - c_o - \frac{c_m}{L}\big)}{\big(P_{\max} + \eta\cdot c_g\big)\big(\beta L + w\cdot c_m\big(L - \frac{c_m}{P_{\max}+\eta\cdot c_g - c_o}\big)\big)}$$

□

Lemma 7.

$$\text{Cost}^{\text{ty-1}}(y_{\text{OFA}}) \ge m_1\beta + \sum_{k=1}^{m_1}\Big(\frac{(q_i^1+\beta)(Lc_o + c_m)}{L\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m}$$
$$w\cdot c_m + \frac{c_o(-q_i^1 + w\cdot c_m)}{P_{\max} + \eta\cdot c_g - c_o}\Big) \tag{105}$$

Proof. Consider a particular type-1 segment $[T_i^c+1, T_{i+1}^c]$. Denote the costs of y_{OFA} during $[T_i^c+1, \tilde{T}_i^c - w - 1]$ and $[\tilde{T}_i^c - w, T_{i+1}^c]$ by Cost^{up} and Cost^{pt} respectively.

Step 1: We bound Cost^{up} as follows:

$$\text{Cost}^{\text{up}} = \beta + \sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1} \psi\big(\sigma(t),1\big) \tag{106}$$
$$= \beta + (\tilde{T}_i^c - w - 1 - T_i^c)c_m + \sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\psi\big(\sigma(t),1\big) - c_m\big).$$

On the other hand, we obtain

$$\sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\psi\big(\sigma(t),1\big) - c_m\big)$$
$$= \frac{\sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\psi\big(\sigma(t),1\big) - c_m\big)}{\sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\big)}$$
$$\times \sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\big) \tag{107}$$
$$\ge \frac{c_o}{P_{\max} + \eta\cdot c_g - c_o} \tag{108}$$
$$\times \sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\big) \tag{109}$$

The last inequality follows from Lemma 5.

Next, we bound the second term by

$$\sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\big) \tag{110}$$
$$\ge \sum_{t=T_i^c+1}^{\tilde{T}_i^c-w-1}\big(\delta(t) + c_m\big) \tag{111}$$
$$\ge \triangle\big(\tilde{T}_i^c - w - 1\big) - \triangle\big(T_i^c\big) + (\tilde{T}_i^c - w - 1 - T_i^c)c_m$$
$$= q_i^1 + \beta + (\tilde{T}_i^c - w - 1 - T_i^c)c_m \tag{112}$$

Together, we obtain

$$\text{Cost}^{\text{up}}$$
$$\ge \beta + (\tilde{T}_i^c - w - 1 - T_i^c)c_m + \tag{113}$$
$$\frac{c_o}{P_{\max} + \eta\cdot c_g - c_o}\Big(q_i^1 + \beta + (\tilde{T}_i^c - w - 1 - T_i^c)c_m\Big) \tag{114}$$
$$= \beta + \frac{(q_i^1+\beta)c_o + (\tilde{T}_i^c - w - 1 - T_i^c)\big(P_{\max} + \eta\cdot c_g\big)c_m}{P_{\max} + \eta\cdot c_g - c_o}$$

Furthermore, we note that $\big(\tilde{T}_i^c - w - 1 - T_i^c\big)$ is lower bounded by the steepest descend when $p(t) = P_{\max}$, $a(t) = L$ and $h(t) = \eta L$,

$$\tilde{T}_i^c - w - 1 - T_i^c \ge \frac{q_i^1 + \beta}{L\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m} \tag{115}$$

By Eqns. (115)-(115), we obtain

$$\text{Cost}^{\text{up}}$$
$$\ge \beta + \frac{(q_i^1+\beta)c_o + (\tilde{T}_i^c - w - 1 - T_i^c)\big(P_{\max} + \eta\cdot c_g\big)c_m}{P_{\max} + \eta\cdot c_g - c_o}$$
$$\ge \beta + \frac{(q_i^1+\beta)(Lc_o + c_m)}{L\big(P_{\max} + \eta\cdot c_g - c_o\big) - c_m} \tag{116}$$

Step 2: We bound $\mathrm{Cost}^{\mathrm{pt}}$ as follows.

$$\mathrm{Cost}^{\mathrm{pt}} = \sum_{t=\tilde{T}_i^c-w}^{T_{i+1}^c} \psi\big(\sigma(t),1\big) \tag{117}$$

$$= (T_{i+1}^c - \tilde{T}_i^c + w + 1)c_m + \sum_{t=\tilde{T}_i^c-w}^{T_{i+1}^c} \big(\psi\big(\sigma(t),1\big) - c_m\big) \tag{118}$$

$$\geq w \cdot c_m + \tag{119}$$

$$\frac{c_o}{P_{\max} + \eta \cdot c_g - c_o} \sum_{t=\tilde{T}_i^c-w}^{T_{i+1}^c} \big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\big)$$

On the other hand, we obtain

$$\sum_{t=\tilde{T}_i^c-w}^{T_{i+1}^c} \big(\psi\big(\sigma(t),0\big) - \psi\big(\sigma(t),1\big) + c_m\big) \tag{120}$$

$$= \sum_{t=\tilde{T}_i^c-w}^{T_{i+1}^c} \delta(t) + (T_{i+1}^c - \tilde{T}_i^c + w + 1)c_m \tag{121}$$

$$\geq \triangle(T_{i+1}^c) - \triangle(\tilde{T}_i^c - w - 1) + w \cdot c_m = w \cdot c_m - q_i^1$$

Therefore,

$$\mathrm{Cost}^{\mathrm{pt}} \geq w \cdot c_m + \frac{c_o(w \cdot c_m - q_i^1)}{P_{\max} + \eta \cdot c_g - c_o} \tag{122}$$

Since there are m_1 type-1 critical segments, according to Eqna. (116)-(122), we obtain

$$\mathrm{Cost}^{\mathrm{ty\text{-}1}}(y_{\mathrm{OFA}}) \tag{123}$$

$$\geq m_1\beta + \sum_{k=1}^{m_1} \Big(\frac{(q_i^1 + \beta)(Lc_o + c_m)}{L\big(P_{\max} + \eta \cdot c_g - c_o\big) - c_m}$$

$$w \cdot c_m + \frac{c_o(-q_i^1 + w \cdot c_m)}{P_{\max} + \eta \cdot c_g - c_o}\Big). \tag{124}$$

□

E. PROOF OF THEOREM 5

Theorem 5. Suppose (y_n, u_n, v_n, s_n) is an optimal offline solution for each $\mathbf{fMCMP}_{\mathrm{s}}^{\mathrm{ly}\text{-}n}$ $(1 \leq n \leq N)$. Then $((y_n^*, u_n^*)_{n=1}^N, v^*, s^*)$ defined as follows is an optimal offline solution for **fMCMP**:

$$\begin{aligned} y_n^*(t) &= y_n(t), \quad v^*(t) = a^{\mathrm{top}}(t) + \textstyle\sum_{n=1}^N v_n(t) \\ u_n^*(t) &= u_n(t), \quad s^*(t) = h^{\mathrm{top}}(t) + \textstyle\sum_{n=1}^N s_n(t) \end{aligned} \tag{125}$$

Proof. First, since that $(a^{\mathrm{top}}, h^{\mathrm{top}})$ can only be satisfied from external supplies, we can assume $a^{\mathrm{top}}(t) = 0, h^{\mathrm{top}}(t) = 0$ for all t without loss of generality.

We then present the basic idea of proof. Suppose that $((\tilde{y}_n, \tilde{u}_n)_{n=1}^N, \tilde{v}, \tilde{s})$ is an optimal solution for **fMCMP**. We will show that we can construct a new feasible solution $((\widehat{y}_n, \widehat{u}_n)_{n=1}^N, \widehat{v}, \widehat{s})$ for **fMCMP**, and a new feasible solution $(\widehat{y}_n, \widehat{u}_n, \widehat{v}_n, \widehat{s}_n)$ for each $\mathbf{fMCMP}_{\mathrm{s}}^{\mathrm{ly}\text{-}n}$, such that

$$\mathrm{Cost}(\tilde{y}, \tilde{u}, \tilde{v}, \tilde{s}) \geq \mathrm{Cost}(\widehat{y}, \widehat{u}, \widehat{v}, \widehat{s}) \geq \sum_{n=1}^N \mathrm{Cost}(\widehat{y}_n) \tag{126}$$

where the second inequality follows from Lemma 1.

(y_n, u_n, v_n, s_n) is an optimal solution for each $\mathbf{fMCMP}_{\mathrm{s}}^{\mathrm{ly}\text{-}n}$. Hence, $\mathrm{Cost}(\widehat{y}_n) \geq \mathrm{Cost}(y_n)$ for each n. Thus,

$$\begin{aligned} \mathrm{Cost}(\tilde{y}, \tilde{u}, \tilde{v}, \tilde{s}) \geq \sum_{n=1}^N \mathrm{Cost}(\widehat{y}_n) &\geq \sum_{n=1}^N \mathrm{Cost}(y_n^*) \\ &= \mathrm{Cost}(y^*, u^*, v^*, s^*) \end{aligned} \tag{127}$$

Hence, $((y_n^*, u_n^*)_{n=1}^N, v^*, s^*)$ is an optimal solution for **fMCMP**.

It only remains to prove Eqn (126). Define $(\widehat{y}_n, \widehat{u}_n)$ based on $(\tilde{y}_n, \tilde{u}_n)$ by:

$$\widehat{y}_n(t) = \begin{cases} 1, & \text{if } n \leq \sum_{r=1}^N \tilde{y}_r(t) \\ 0, & \text{otherwise} \end{cases} \tag{128}$$

and

$$\begin{aligned} \widehat{u}_1(t) &= \min\{L, \textstyle\sum_{r=1}^N \tilde{u}_r(t)\} \\ \widehat{u}_n(t) &= \min\{L, \textstyle\sum_{r=1}^N \tilde{u}_r(t) - \sum_{r=1}^{n-1} \tilde{u}_r(t)\} \end{aligned} \tag{129}$$

Note that we have sliced $\sum_{r=1}^N \tilde{u}_r(t)$ into N layers $(\widehat{u}_n(t))_{n=1}^N$ in the same manner as $(a^{\mathrm{ly}\text{-}n}(t))_{n=1}^N$.

It is straightforward to see that

$$\sum_{r=1}^N \tilde{y}_r(t) = \sum_{r=1}^N \hat{y}_r(t) \text{ and } \sum_{r=1}^N \tilde{u}_r(t) = \sum_{r=1}^N \hat{u}_r(t) \tag{130}$$

Furthermore, we define

$$\widehat{v}_n(t) = \big[a^{\mathrm{ly}\text{-}n}(t) - \widehat{u}_n(t)\big]^+, \; \widehat{s}_n(t) = \big[h^{\mathrm{ly}\text{-}n}(t) - \eta \cdot \widehat{u}_n(t)\big]^+$$

It follows that $(\widehat{y}_n, \widehat{u}_n, \widehat{v}_n, \widehat{s}_n)$ is feasible for $\mathbf{fMCMP}_{\mathrm{s}}^{\mathrm{ly}\text{-}n}$.

Finally, by Lemmas 8-9, we can show that

$$\mathrm{Cost}(\tilde{y}, \tilde{u}, \tilde{v}, \tilde{s}) \geq \mathrm{Cost}(\widehat{y}, \widehat{u}, \widehat{v}, \widehat{s}) \tag{131}$$

Therefore, it completes the proof. □

Lemma 8.

$$\sum_{r=1}^N \widehat{v}_r(t) \leq \sum_{r=1}^N \tilde{v}_r(t) = \tilde{v}(t) \tag{132}$$

$$\sum_{r=1}^N \widehat{s}_r(t) \leq \sum_{r=1}^N \tilde{s}_r(t) = \tilde{s}(t) \tag{133}$$

Proof. First, we note that $((\tilde{y}_n, \tilde{u}_n)_{n=1}^N, \tilde{v}, \tilde{s})$ is an optimal solution for **fMCMP**. Hence, $a(t) \geq \sum_{r=1}^N \tilde{u}_r(t)$ for all t. Then,

$$a^{\mathrm{ly}\text{-}n}(t) \geq \widehat{u}_n(t) \tag{134}$$

Because we slice $\sum_{r=1}^N \tilde{u}_r(t)$ into N layers $(\widehat{u}_n(t))_{n=1}^N$ in a same manner as $a(t)$ into $(a^{\mathrm{ly}\text{-}n}(t))_{n=1}^N$. Hence,

$$\sum_{r=1}^N \widehat{v}_r(t) = \sum_{r=1}^N \big[a^{\mathrm{ly}\text{-}r}(t) - \widehat{u}_r(t)\big]^+ \tag{135}$$

$$= \sum_{r=1}^N \Big(a^{\mathrm{ly}\text{-}r}(t) - \widehat{u}_r(t)\Big) = a(t) - \sum_{r=1}^N \widehat{u}_r(t) \tag{136}$$

$$= a(t) - \sum_{r=1}^N \tilde{u}_r(t) \leq \sum_{r=1}^N \tilde{v}_r(t) = \tilde{v}(t) \tag{137}$$

where the last inequality follows constraint (C'4).

Similarly, we can prove Eqn. (133) considering (C'5). □

LEMMA 9.

$$\sum_{r=1}^{N}[\widehat{y}_r(t)-\widehat{y}_r(t\text{-}1)]^+ \le \sum_{r=1}^{N}[\tilde{y}_r(t)-\tilde{y}_r(t\text{-}1)]^+ \tag{138}$$

PROOF. First, note that $\widehat{y}_1(t) \ge ... \ge \widehat{y}_N(t)$ is a decreasing sequence. Because $\widehat{y}_n(t) \in \{0,1\}$ for all n, we obtain

$$\sum_{r=1}^{N}[\widehat{y}_r(t)-\widehat{y}_r(t\text{-}1)]^+ \tag{139}$$

$$= \begin{cases} 0, & \text{if } \sum_{r=1}^{N}\widehat{y}_r(t) \ge \sum_{r=1}^{N}\widehat{y}_r(t\text{-}1) \\ \sum_{r=1}^{N}\widehat{y}_r(t)-\sum_{r=1}^{N}\widehat{y}_r(t\text{-}1), & \text{otherwise}\end{cases} \tag{140}$$

$$= \Big[\sum_{r=1}^{N}\widehat{y}_r(t)-\sum_{r=1}^{N}\widehat{y}_r(t\text{-}1)\Big]^+ \tag{141}$$

$$= \Big[\sum_{r=1}^{N}\tilde{y}_r(t)-\sum_{r=1}^{N}\tilde{y}_r(t\text{-}1)\Big]^+ \tag{142}$$

$$\le \Big[\sum_{r=1}^{N}[\tilde{y}_r(t)-\tilde{y}_r(t\text{-}1)]^+\Big]^+ = \sum_{r=1}^{N}[\tilde{y}_r(t)-\tilde{y}_r(t\text{-}1)]^+$$

□

F. PROOF OF THEOREM 7

Theorem 7. The competitive ratio of $\mathsf{CHASE}_{\mathrm{gen}}^{\mathrm{lk(w)}}$ is upper bounded by $r_1(w)\cdot\max\left(r_2,r_3\right)$, where

$$r_1(w) = 1+\frac{2\beta L\left(P_{\max}+\eta\cdot c_g-c_o-\frac{c_m}{L}\right)}{\left(P_{\max}+\eta\cdot c_g\right)\left(\beta L+w\cdot c_m\left(L-\frac{c_m}{P_{\max}+\eta\cdot c_g-c_o}\right)\right)}$$

$$r_2 = 1+\max\left\{\frac{\left(P_{\max}+c_g\cdot\eta-c_0\right)}{Lc_0+c_m}\max\left\{0,\left(L-\mathsf{R}_{\mathrm{up}}\right)\right\}\right. \left.\frac{c_o}{c_m}\max\left\{0,\left(L-\mathsf{R}_{\mathrm{dw}}\right)\right\}\right\}$$

$$r_3 = 1+\frac{L\left(P_{\max}+c_g\cdot\eta\right)+c_m}{\beta}\mathsf{T}_{\mathrm{on}}+\frac{L\left(P_{\max}+c_g\cdot\eta\right)}{\beta}\mathsf{T}_{\mathrm{off}}$$

PROOF. The competitive ratio can be expressed as follows:

$$\begin{aligned}\mathsf{CR}(\mathsf{CHASE}_{gen}) &= \max_{\sigma}\frac{\mathrm{Cost}_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}_{\mathsf{Opt}}(\sigma)} \\ &\le \max_{\sigma}\frac{\mathrm{Cost}_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}_{\mathsf{iOpt}}(\sigma)} \\ &= \max_{\sigma}\frac{\mathrm{Cost}_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}_{\mathsf{CHASE}_s}(\sigma)}\cdot\frac{\mathrm{Cost}_{\mathsf{CHASE}_s}(\sigma)}{\mathrm{Cost}_{\mathsf{iOpt}}(\sigma)}\end{aligned} \tag{143}$$

In Eqn. (143), we have the following notations:

- $\mathrm{Cost}_{\mathsf{CHASE}_{gen}}(\sigma)$: The cost of online algorithm $\mathsf{CHASE}_{\mathrm{gen}} = \left(y_2,u_2,v_2,s_2\right)$ for **MCMP** with input σ
- $\mathrm{Cost}_{\mathsf{Opt}}(\sigma)$: The cost of offline optimal algorithm for **MCMP** with input σ
- $\mathrm{Cost}_{\mathsf{iOpt}}(\sigma)$: The cost of offline optimal algorithm for **fMCMP** with input σ
- $\mathrm{Cost}_{\mathsf{CHASE}_s}(\sigma)$: The cost of online algorithm CHASEs $= \left(y_s,u_s,v_s,s_s\right)$ for **fMCMP** with input σ

Now, we analyze each term in Eqn. (143).

First, as we prove in Theorem 2,

$$\begin{aligned}&\frac{\mathrm{Cost}_{\mathsf{CHASE}_s}(\sigma)}{\mathrm{Cost}_{\mathsf{iOpt}}(\sigma)} \\ \le\ & 1+\frac{2\beta L\left(P_{\max}+\eta\cdot c_g-c_o-\frac{c_m}{L}\right)}{\left(P_{\max}+\eta\cdot c_g\right)\left(\beta L+w\cdot c_m\left(L-\frac{c_m}{P_{\max}+\eta\cdot c_g-c_o}\right)\right)} \\ &= r_1(w)\end{aligned}$$

Second, upper-bounding the term $\frac{\mathrm{Cost}_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}_{\mathsf{CHASE}_s}(\sigma)}$ We divide the cost of CHASE_s into two parts:

$$\mathrm{Cost}_{\mathsf{CHASE}_s}(\sigma) = \mathrm{Cost}^e_{\mathsf{CHASE}_s}(\sigma)+\mathrm{Cost}^n_{\mathsf{CHASE}_s}(\sigma)$$

where:

$$\mathrm{Cost}^e_{\mathsf{CHASE}_s}(\sigma) = \sum_{t\in\mathbf{T_e}} c_o u_s(t)+p(t)v_s(t)+c_g s_s(t)+c_m y_s(t)$$

$$\begin{aligned}\mathrm{Cost}^n_{\mathsf{CHASE}_s}(\sigma) &= \sum_{t\in\mathbf{T_n}} c_o u_s(t)+p(t)v_s(t)+c_g s_s(t)+c_m y_s(t) \\ &\quad+\sum_{t=1}^{T}\beta\left[y_s(t)-y_s(t-1)\right]^+.\end{aligned}$$

and $\mathbf{T_e} = \{t|y_s(t)=y_2(t)\}$, $\mathbf{T_n} = \{t|y_s(t)\neq y_2(t)\}$

Similarly, we divide the cost of $\mathsf{CHASE}_{\mathrm{gen}}$ into two parts:

$$\mathrm{Cost}_{\mathsf{CHASE}_{gen}}(\sigma) = \mathrm{Cost}^e_{\mathsf{CHASE}_{gen}}(\sigma)+\mathrm{Cost}^n_{\mathsf{CHASE}_{gen}}(\sigma)$$

Therefore,

$$\frac{\mathrm{Cost}_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}_{\mathsf{CHASE}_s}(\sigma)} \le \max\left(\frac{\mathrm{Cost}^e_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}^e_{\mathsf{CHASE}_s}(\sigma)},\frac{\mathrm{Cost}^n_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}^n_{\mathsf{CHASE}_s}(\sigma)}\right)$$

Next we will prove

$$\frac{\mathrm{Cost}^e_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}^e_{\mathsf{CHASE}_s}(\sigma)} \le r_2$$

Note that

$$\begin{aligned}&\frac{\mathrm{Cost}^e_{\mathsf{CHASE}_{gen}}(\sigma)}{\mathrm{Cost}^e_{\mathsf{CHASE}_s}(\sigma)} \\ =\ & \frac{\sum_{t\in\mathbf{T_e}}\left(c_o u_2(t)+p(t)v_2(t)+c_m y_2(t)+c_g s_2(t)\right)}{\sum_{t\in\mathbf{T_e}}\left(c_o u_s(t)+p(t)v_s(t)+c_m y_s(t)+c_g s_s(t)\right)} &(144) \\ \le\ & \max_{\forall t\in\mathbf{T_e}}\frac{c_o u_2(t)+p(t)v_2(t)+c_m y_2(t)+c_g s_2(t)}{c_o u_s(t)+p(t)v_s(t)+c_m y_s(t)+c_g s_s(t)} &(145)\end{aligned}$$

(145) says to build a upper bound of $\mathrm{Cost}^e_{\mathsf{CHASE}_{gen}}(\sigma)/\mathrm{Cost}^e_{\mathsf{CHASE}_s}(\sigma)$ over time slots, we only need to consider the maximum ratio on a single time slot. When $y_2(t)=y_s(t)=0$, it is easy to see $u_2(t)=u_s(t)=0,\ v_2(t)=v_s(t),\ s_2(t)=s_s(t) \rightarrow \mathrm{Cost}^e_{\mathsf{CHASE}_{gen}}(\sigma)/\mathrm{Cost}^e_{\mathsf{CHASE}_s}(\sigma)=1$ Thus, we only consider the situation when $y_2(t)=y_s(t)=1$:

Case 1: $u_s(t)<u_2(t-1)$ For $\mathsf{CHASE}_{\mathrm{gen}}$:

$$\begin{aligned}u_2(t) &= u_2(t-1)-\min\left(\mathsf{R}_{\mathrm{dw}},u_2(t-1)-u_s(t)\right) \\ &= \max\{u_2(t-1)-R_{dw},u_s(t)\} \\ &\le \max\{L-R_{dw},u_s(t)\}\end{aligned}$$

Therefore,

$$\begin{aligned}
& \frac{c_o u_2(t) + p(t)v_2(t) + c_m y_2(t) + c_g s_2(t)}{c_o u_s(t) + p(t)v_s(t) + c_m y_s(t) + c_g s_s(t)} \\
= & \frac{c_o u_2(t) + p(t)v_2(t) + c_g s_2(t) + c_m}{c_o u_s(t) + p(t)v_s(t) + c_g s_s(t) + c_m} && (146) \\
\leq & 1 + \frac{c_o\big(u_2(t) - u_s(t)\big)}{c_m + c_o u_s(t)} \\
& \frac{p(t)\big(v_2(t) - v_s(t)\big) + c_g\big(s_2(t) - s_s(t)\big)}{c_m + c_o u_s(t)} && (147) \\
\leq & 1 + \frac{c_o \max\{L - R_{dw} - u_s(t), 0\}}{c_m + c_o u_s(t)} && (148) \\
= & 1 + \frac{c_o}{c_m}\max\{L - R_{dw}, 0\} && (149)
\end{aligned}$$

Eqn. (148) is from $v_2(t) \leq v_s(t),\ s_2(t) \leq s_s(t)$, this is because $u_2(t) \geq u_s(t)$

Case 2: $u_s(t) \geq u_2(t-1)$ For $\mathsf{CHASE}_{\text{gen}}$:

$$\begin{aligned}
u_2(t) &= u_2(t-1) + \min\big(\mathsf{R}_{\text{up}}, u_s(t) - u_2(t-1)\big) \\
&= \min\{u_2(t-1) + \mathsf{R}_{\text{up}}, u_s(t)\}
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \frac{c_o u_2(t) + p(t)v_2(t) + c_m + c_g s_2(t)}{c_o u_s(t) + p(t)v_s(t) + c_m + c_g s_s(t)} \\
= & 1 + \frac{c_o\big(u_2(t) - u_s(t)\big)}{c_o u_s(t) + c_m} \\
& \frac{p(t)\big(v_2(t) - v_s(t)\big) + c_g\big(s_2(t) - s_s(t)\big)}{c_o u_s(t) + c_m} && (150) \\
\leq & \frac{\big(p(t) + c_g\eta - c_o\big)\big(u_s(t) - u_2(t)\big)}{c_o u_s(t) + c_m} + 1 && (151) \\
\leq & 1 + \big(P_{\max} + c_g\eta - c_o\big)\frac{\max\big(0, u_s(t) - u_2(t-1) - \mathsf{R}_{\text{up}}\big)}{c_o u_s(t) + c_m} \\
\leq & 1 + \big(P_{\max} + c_g\eta - c_o\big)\frac{\max\big(0, u_s(t) - \mathsf{R}_{\text{up}}\big)}{c_o u_s(t) + c_m} \\
\leq & 1 + \big(P_{\max} + c_g\eta - c_o\big)\frac{\max\big(0, L - \mathsf{R}_{\text{up}}\big)}{L c_o + c_m}
\end{aligned}$$

Eqn. (151) is from $v_2(t) - v_s(t) \leq u_s(t) - u_2(t),\ s_2(t) - s_s(t) \leq \eta \cdot \big(u_s(t) - u_2(t)\big)$ Note now we have $u_s \geq u_2$:

$$\begin{aligned}
v_2 - v_s &= \big[a - u_2\big]^+ - \big[a - u_s\big]^+ \\
&= \max\big(a - u_2, 0\big) + \min\big(u_s - a, 0\big) \\
&= \begin{cases} a - u_2 & if\ u_s > a > u_2 \\ 0 & if\ u_s > u_2 > a \\ u_s - a & if\ a > u_s > u_2 \end{cases} \\
&\leq u_s - u_2
\end{aligned}$$

Similarly we prove $s_2(t) - s_s(t) \leq \eta \cdot \big(u_s(t) - u_2(t)\big)$

Summarizing the above two cases, we conclude that

$$\begin{aligned}
& \frac{\text{Cost}^e_{\mathsf{CHASE}_{gen}}(\sigma)}{\text{Cost}^e_{\mathsf{CHASE}_s}(\sigma)} \\
\leq & 1 + \max\Bigg\{\big(P_{\max} + c_g\eta - c_o\big)\frac{\max\big(0, L - \mathsf{R}_{\text{up}}\big)}{L c_o + c_m}, \\
& \frac{c_o}{c_m}\max\{L - R_{dw}, 0\}\Bigg\}
\end{aligned}$$

Now, we still need to upper bound the term $\frac{\text{Cost}^n_{\mathsf{CHASE}_{gen}}(\sigma)}{\text{Cost}^n_{\mathsf{CHASE}_s}(\sigma)}$ Note the set $\mathbf{T_n}$ represents the time durations when CHASE_{gen} and CHASE_s have different on/off status, and this can only occur on the minimum on/off periods: $\text{T}_{\text{on}}/\text{T}_{\text{off}}$ That is: when CHASE_s has a startup, CHASE_{gen} can not startup the generator because the generator is during its minimum off periods T_{off} Similarly, we know such mismatch of on/off status also occurs during the minimum on periods T_{on} of CHASE_{gen}.

As CHASE_{gen} always follows CHASE_s to startup, thus CHASE_{gen} has at most the same number of startups as CHASE_s. If we denote the number of the startup of $\mathsf{CHASE}_{\text{s}}$ and CHASE_{gen} as k_s and k_2, we have[3]:

$$k_2 \leq k_s$$

Therefore, we can find a lower bound of $\text{Cost}^n_{\mathsf{CHASE}_s}(\sigma)$:

$$\text{Cost}^n_{\mathsf{CHASE}_s}(\sigma) \geq k_s \cdot \beta.$$

We cam also find an upper bound of $\text{Cost}^n_{\mathsf{CHASE}_2}(\sigma)$, by assuming on the minimum on/off periods incurring the maximum cost:

$$\begin{aligned}
& \text{Cost}^n_{\mathsf{CHASE}_2}(\sigma) \\
\leq & k_2\big(\beta + T_{on}(LP_{\max} + \eta c_g L + c_m) + \mathsf{T}_{\text{off}}L\big(P_{\max} + \eta c_g\big)\big)
\end{aligned}$$

Thus,

$$\begin{aligned}
& \frac{\text{Cost}^n_{\mathsf{CHASE}_2}(\sigma)}{\text{Cost}^n_{\mathsf{CHASE}_s}(\sigma)} \\
\leq & \frac{k_2\big(\beta + T_{on}(LP_{\max} + \eta c_g L + c_m) + \mathsf{T}_{\text{off}}L\big(P_{\max} + \eta c_g\big)\big)}{k_s \cdot \beta} \\
\leq & 1 + \frac{T_{on}(LP_{\max} + \eta c_g L + c_m) + \mathsf{T}_{\text{off}}L\big(P_{\max} + \eta c_g\big)}{\beta} \\
= & r_3
\end{aligned}$$

It completes the proof. □

[3]Without loss of generality, we assume $k_s \geq 1$ Otherwise, $k_s = 0$, then $k_2 = 0$, thus $\text{Cost}^n_{\mathsf{CHASE}_{gen}}(\sigma)/\text{Cost}^n_{\mathsf{CHASE}_s}(\sigma) = 1$